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ABSTRACT

This kit is designed to develop the adult numeracy skills of literacy and mathematics trained teachers in Australia using a participatory workshop approach. The kit is divided into five chapters. Each chapter contains theoretical and background information, a sequence of structured activities in which participating teachers actively engage, and materials for them to use with their own students. All activities have been tested with groups of teachers to ensure that they succeed with their target audience. Chapter 1 provides detailed guidelines for planning and conducting a successful staff development activity, from the initial publicity to the final evaluation of the program. Chapter 2 examines the basic mathematics skills needed to function effectively in one's daily life, at home and at work, including measurement, number skills, problem solving, using a calculator, and estimation. Chapter 3 takes up issues of confidence and fear of mathematics and provides guidelines for building a positive attitude to the subject, both for teachers themselves and their students. Chapter 4 examines more closely the teaching strategies themselves, including the following: teaching in context, cooperative learning, the use of hands-on materials and games, as well as strategies for working with mixed ability groups. Chapter 5 examines the relationship between language and mathematics and provides guidelines for successfully integrating literacy and numeracy in the classroom. Contains five references; some chapters also contain references. (YLB)

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BREAKING THE MATHS BARRIER

A Kit for Building Staff Development Skills in Adult Numeracy

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BREAKING THE MATHS BARRIER

A Kit for Building Staff Development Skills in Adult Numeracy

Beth Marr
and
Sue Helme

DEET

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About the Authors

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Beth has been teaching mathematics in a variety of settings, secondary tertiary and TAFE for almost twenty years. During much of this time she has been an active feminist, involved in the establishment of Melbourne's first women's refuge, and printing and publishing cooperative. For the last six years she has combined her interests in mathematics teaching, feminism and publishing by developing and establishing a maths and science bridging course for adult women, and coordinating the Teaching Maths to Women Project. This project brought together many teachers of adults to produce the teacher resource, *Mathematics: a New Beginning*, and conducted staff development for other mathematics teachers.

More recently, Beth has been working with Sue Helme and other colleagues, presenting staff development activities for teachers of basic mathematics and numeracy for adults, and on the production of an adult numeracy book, *Strength in Numbers*. She is currently working half time in the Victorian Division of Further Education as a Numeracy Curriculum Officer and teaching at Northern Metropolitan College of TAFE.

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Sue came to TAFE teaching in 1981, from a background in psychology, youth work and photography. Working with students in the Footscray Womens Learning Centre, she realised the importance of making mathematics more relevant and accessible to adult learners, particularly women, and developed strategies to assist students to overcome their fears of maths and develop more confidence in their ability to learn. Sue has worked with Beth on The Teaching Maths To Women Project and in many in-service programs for adult numeracy teachers in Victoria and interstate. She is currently teaching at Broadmeadows TAFE College in Victoria and is involved in post graduate studies in mathematics education.

Dave Tout

Dave started his educational career as a secondary school maths and science teacher in Victoria. After a period of teaching in the UK he returned to Australia and started teaching in TAFE colleges. This included working with adults in numeracy and basic mathematics. In Victoria, Dave has played a major role in establishing the importance of mathematics in adult numeracy and basic education. He has worked as a consultant in the area of basic maths and numeracy and professional development, and is currently working for the Division of Further Education, helping to run the Adult Basic Education Resource and Information Service (ARIS) for Victoria.

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A number of activities included in the kit are variations of those used by colleagues or modifications of ideas that have appeared in mathematics education publications. Other ideas have come from mathematics educators who have provided inspiration in their workshops. To all these people, our sincere thanks.

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- Teaching Maths to Women Project for the worksheet ***Where are the Decimal Points?*** and ***Fraction Circles;***
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Contents

Introduction 1

Chapter 1. Designing and Running a Numeracy Staff Development Program. 5

- How the Kit is Structured 6
- Planning the Workshop 7
- Preliminary Organisation 8
- Structuring the Workshop 9
- How to Start your Workshop 10
- Evaluating your Program 11

Chapter 2. What do We Teach? 21

- **What is Numeracy? (Background Notes)** 22
  - Activity 2.1 Startling Statements 25
  - Activity 2.2 Maths Diary 36
- **Measurement and the Metric System (Background Notes)** 40
  - Activity 2.3 Metric Estimation 45
- **Space and Shape (Background Notes)** 59
  - Activity 2.4 Pentominoes 62
  - Activity 2.5 Creative Use of the Street Directory 67
- **Number Skills (Background Notes)** 76
  - Activity 2.6 *Real Maths, School Maths:* A Discussion of the Video 80
- **Calculators and Adult Numeracy (Background Notes)** 86
  - Activity 2.7 Using Calculators Effectively 90
- **Problem Solving (Background Notes)** 96
  - Problem Solving for Numeracy Students 104

Chapter 3. Building Positive Attitudes. 129

- **Building Positive Attitudes (Background Notes)** 130
  - Activity 3.1 Sharing Experiences of Maths 134
  - Activity 3.2 How Adults Learn 145
  - Activity 3.3 Experiencing Success with Metric Length 149
  - Activity 3.4 Acknowledging Students' Skills and Experiences 156

Chapter 4. Teaching Strategies. 161

- **Teaching in Context (Background Notes)** 162
  - Activity 4.1 Maths on the Supermarket Shelves 165
  - Activity 4.2 Newspaper Numeracy 169
- **Using Hands on Materials (Background Notes)** 174
  - Activity 4.3 Modelling Place Value, Addition and Subtraction 179
  - Activity 4.4 Problem Solving Stations 188
  - Activity 4.5 Experiencing Fractions 205
- **Interaction and Cooperation (Background Notes)** 219
  - Activity 4.6 Cooperative Logic 229
  - Activity 4.7 Exploring Groupwork with Problem Solving 242
- **Using Games in Adult Numeracy (Background Notes)** 259
  - Activity 4.8 How Close can You Get? 261
  - Activity 4.9 Multidigit 264
  - Activity 4.10 Digititis 264
- **Meeting Individual Needs (Background Notes)** 271
  - Activity 4.11 Meeting Individual Needs 273

Chapter 5. Language and Maths. 277

- **Language and Maths (Background Notes)** 278
  - Activity 5.1 Integrating Literacy and Numeracy 288
  - Activity 5.2 Words for the Four Operations 292
  - Activity 5.3 Mathematical Talk 297
  - Activity 5.4 Maths Words 299

Introduction

Adult basic education has a fundamental role to play in ensuring access to further education, training and meaningful work. It is a field in which the Government's social justice and economic strategies find common ground. Ways and means need to be provided for our workforce to become better skilled and more flexible. At the same time, the principles of social justice must be translated to practice, to ensure that people who missed out on acquiring basic education skills the first time around get a second chance.

To participate effectively in today's advanced technological society, it is necessary to comprehend and critically evaluate large amounts of information, much of which is presented in numerical or graphical form. For example, to interpret information when listening to radio, watching television, reading newspapers or magazines, or understanding diagrams or instructions in the workplace, a range of mathematical skills are needed. These include basic number skills, spatial concepts and logical problem solving.

Yet even simple numeracy tasks are beyond the reach of many adults in Australia. A recent national survey on adult literacy (Wickert, 1989) found a significantly low level of basic mathematical competence in the adult population. The survey found that 23% of the total sample could not correctly total two entries on a bank deposit slip; 38% could not work out the change from $5 after ordering a sandwich and soup from a menu; 45% could not enter two withdrawals and two deposits in a bank record book and keep the running total; and 57% could not calculate a 10% surcharge on a restaurant bill after ordering two items totalling less than $5. The survey interviewed a sample of 1500 adults aged 18 and over from all States, not living in institutions.

There was a strong correlation between literacy and numeracy skills, leading Wickert to conclude:

> *It is clear from the results that when people have poor literacy skills, they have even worse numeracy skills. The need to upgrade numeracy skills in the context of literacy must be taken into account in all decisions to raise the level of adult literacy on Australia (p. xii).*

A recent survey of adult literacy students in New South Wales (Allen and Lord, 1991) confirmed these results. 56% of the literacy students interviewed said they could not manage to work out on their own the cost of an order at a restaurant, pub or takeaway, and 50% said they could not work out the cost of an item discounted 25%.

In the light of these findings it is clear that the numeracy needs of adult basic education students must be addressed.

What is Numeracy?

It would be helpful to provide universally agreed upon definitions of numeracy and mathematics and clearly delineate how these differ. However up to now no such formal definitions exist. We can however, offer some of the working definitions currently in use, in order to give a general feeling of what is meant by the word.

We begin with a brief statement from the introduction to a new Australian book about Numeracy issues:

> *To be numerate is to function effectively mathematically in one's daily life, at home and at work* (Willis, 1990).

The Beazley committee from Western Australia defined numeracy as:

> *the mathematics for effective functioning in one's group and community, and the capacity to use these skills to further one's own development and that of one's community* (Beazley, 1984).

From the UK, one of the most famous and often quoted reports on mathematics education, the Cockroft report (1982) says:

> *We would wish the word 'numerate' to imply the possession of two attributes. The first is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enable an individual to cope with the practical demands of his everyday life. The second is an ability to have some apprehension and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease. Taken together these imply that a numerate person should be expected to be able to appreciate and understand some of the ways in which mathematics can be used as a means of communication ... Our concern is that those who set out to make pupils 'numerate' should pay attention to the wider aspects of numeracy and not be content merely to develop skills of computation....* (Paragraph 39).
>
> *Most important of all is the need to have sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed, whether this be little or much* (Paragraph 34).

At the time of the writing of this publication a national working group of TAFE teachers in Australia has offered for comment the definition:

> *Numeracy is the understanding and application of the mathematics that a person requires for work, study and everyday life.*
>
> *Numeracy comprises survival skills and numerical and spatial abilities including the use of calculators, estimation, appreciation of shapes, size, direction and measurement, problem solving and logical reasoning, and the interpretation and language of mathematical data and information.*

Each of the definitions presented adds something to an overall picture of numeracy. Firstly, it does not merely refer to operating on numbers, as the word suggests. It is used to represent a much wider range of skills.

In addition a feeling of flexibility emerges: numeracy does not have a clear beginning and end but depends upon the needs and interests of the individual, within the context of their own peer group, community and occupation.

Finally, to complete the picture we have Cockroft's references to attitudes as well as skills.

By referring to 'at homeness' with numbers and 'sufficient confidence' this definition invites the teacher to concentrate on ensuring that when numeracy students walk away from class they are equipped with skills that can actually be put into practice and the freedom from anxiety which will enable them to do so.

The Need for Staff Development

In order to expand provision in adult numeracy, and ensure the quality of that provision, appropriately trained teachers and tutors are required.

Literacy teachers, although not formally trained in mathematics, can contribute significantly to this need by offering numeracy classes, or by incorporating numeracy into their literacy classes, in contexts relevant to their students. Their main need is to recognise the numeracy skills they already possess, build on these skills and develop teaching methodologies and the confidence to tackle the teaching of numeracy.

On the other hand, mathematics trained teachers are looking for assistance in developing appropriate teaching strategies and materials for adult numeracy students. Traditional methods need to be supplemented with alternative approaches, such as hands-on experiences, cooperative learning, and problem solving within contexts relevant to adult learners.

The purpose of this kit is to develop the skills of both groups of teachers. A participatory workshop approach has been chosen because it models the good teaching practice the kit is designed to develop. This approach reflects the following principles:

- learning best takes place through activities with purpose and relevance to the participants;
- sharing good practice by example is far more effective that simply telling participants what to do ('show - not tell');
- participants gain most by reflecting on their practice and, through interaction, learning from each other;
- participants who experience new activities and methods for themselves are more likely to appreciate their value and put them into practice with their own students.

The Contents of this Kit

The kit is divided into five chapters, each of which deals with a separate theme. Each chapter contains theoretical and background information, a sequence of structured activities in which participating teachers actively engage, and materials for them to use with their own students.

All the activities in the kit have been trialled with groups of teachers before being finalised, to ensure that the activities will succeed with their target audience.

Chapter 1: Designing and Running a Numeracy Staff Development Program provides detailed guidelines for planning and conducting a successful staff development activity, from the initial publicity through to the final evaluation of the program.

Chapter 2: What Do We Teach? examines the basic mathematics skills needed to function effectively in one's daily life, at home and at work, including measurement, number skills, problem solving, using a calculator and estimation.

Chapter 3: Building Positive Attitudes takes up the issues of confidence and fear of mathematics, and provides guidelines for building a positive attitude to the subject, both for teachers themselves and their students.

Chapter 4: Teaching Strategies examines more closely the teaching strategies themselves, including teaching in context, cooperative learning, the use of hands-on materials and games, as well as strategies for working with mixed ability groups.

Chapter 5: Language and Maths examines the relationship between language and mathematics and provides guidelines for successfully integrating literacy and numeracy in the classroom.

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Allan, L. and Lord, S. (1991) *For a Number of Reasons*. NSW: Adult Literacy Information Office.

Beazley, K. (1984) *Education in Western Australia: Report of the Committee of Inquiry into Education in Western Australia*. Education Department of Western Australia.

Cockroft, W. H. (Chairman) (1982): *Report of the Committee of Inquiry into the Teaching of Mathematics in Schools*. HMSO, London UK.

Wickert, R. (1989), *No Single Measure: A Survey of Australian Adult Literacy, Summary Report*. Department of Employment, Education and Training, Canberra.

Willis, S. (1990) *Being Numerate: What Counts* ? ACER. Hawthorn Vic.

Further Reading

Other interesting readings to supplement this kit may be found in the publication *Reclaiming Mathematics: the Practice and Role of Numeracy in Society* (1992) eds. Betty Johnston and Vicki Webber, University of Technology, Sydney. This is a collection of in depth articles on the social and theoretical aspects of numeracy teaching, mainly drawn from overseas publications. Sections of the book have been arranged so as to correspond with the chapters of this kit.

Chapter 1. Designing andRunning a Numeracy Staff Development Program.

This chapter provides guidelines for planning and implementing your Numeracy Staff Development Workshops. It includes detailed information on the structure of the kit and guidelines for putting into practice the information it contains.
The kit uses the **Participatory Workshop** model of Staff Development, which **involves participants actively** in learning and information sharing. We have selected this approach because:

- workshops create an environment where participants can question, discuss and think about their methods of teaching, as well as experience and reflect on new approaches to learning and teaching;
- participants who experience new activities and methods for themselves are more likely to appreciate their value and put them into practice with their own students;
- workshops affirm and build on classroom experiences (through interacting with each other participants can recognise the problems and issues they have in common and begin to explore possible solutions);
- workshops enable participants to contribute their ideas and experience and learn from each other.

Experienced workshop leaders may not need to read this chapter in detail. However, it is essential to read Section 1, which explains the structure of the kit, and Section 5, which provides guidelines for selecting appropriate activities for commencing a numeracy workshop. Workshop leaders who are starting out for the first time will find in this chapter clear guidelines for organising, structuring, and evaluating a numeracy staff development activity.

1. How the Kit is Structured

The kit is designed so that an experienced basic education teacher can pick it up and use it with a group of teachers and tutors who wish to build their confidence and skills in teaching numeracy. It is self-contained, providing enough background information to conduct a successful workshop without the need to search for extra references and resources. The background information and philosophical framework have been developed by researchers and experienced practitioners and the activities have all been refined through trialling with groups of teachers at staff development activities.

Background Notes

Each chapter starts with an introduction, which provides background information and some chapters are divided into distinct sections which begin with specific background notes relevant to the topic under discussion. You may choose to photocopy and distribute these background information sections for participants to read at the end of the workshops. The end of the photocopiable section is denoted by a line across the page. Below this line there are descriptions of the activities contained in the chapter or section and sometimes further instructions for presenting the activities.

The Activities

Following the introduction is a series of activities **in which participants actively engage**.

- Each activity may demonstrate a particular method of teaching or generate discussion about an issue.
- All activities are designed to model good teaching practice.
- The kit is designed so that you select activities which are most appropriate to the needs of your group and can be completed within the time available.
- The activities are not intended to be conducted in exactly the order in which they appear in the kit.
- It is not expected that you use **every** activity.

The activities have the following components:

Rationale: This explains why the activity is included, what mathematical skills it explores or develops, and any other relevant background information.

Aims: A brief statement of the skills and knowledge participants are expected to gain from the activity.

Handouts: Includes worksheets to be used by participants during the activity, student activity sheets, important information, summaries, or background reading to supplement the workshop discussion.

Materials and Equipment: Lists all the materials and equipment needed to conduct the activity. The equipment needed has intentionally been limited to cheap and readily accessible items.

If you have access to an overhead projector, you may wish to make overheads from the summaries or worksheets to guide a group discussion. Overheads are more helpful with larger groups. (In small groups they can be alienating and create division between the presenter and the rest of the group.)

Time: The estimated time needed to complete the activity.

Preparation: Provides instructions on materials to collect and/or photocopy, and equipment to organise. The equipment and materials needed have been made as simple and inexpensive as possible to enable sessions to be conducted in any setting. When reference is made to white-boards or overhead projectors, a piece of butchers' paper attached to a wall will suffice.

Conducting the activity: Gives step by step instructions for conducting the activity, including suggested discussion questions. The activities have been carefully developed from previously conducted workshops and can therefore be followed as they are written, but feel free to include ideas and adaptations of your own.

2. Planning the Workshop

Working with a partner

We strongly recommend that you share the planning and facilitating with a colleague. In the planning phase, teamwork provides both moral support and the opportunity to combine the ideas and resources of two or more people, ensuring a better quality program. During the activity, while one person is facilitating, the other can assist with routine tasks such as organising equipment and handing out sheets. This person can also observe and assist participants, and look out for issues for later discussion, while allowing the facilitator to concentrate fully on conducting the session. Roles can then be switched between the session leaders. When both partners feel free to comment tactfully, the mutual support they give ensures that the session is lively, natural and productive. After the workshop the value of working in a team is most evident. It is very useful to complete the evaluation process with your partner, sharing your perceptions of how it all went and giving each other feedback as you identify successful and less successful aspects and plan modifications to your program. Reading and discussing participants' evaluations is also part of this process.

Who is the workshop for?

Define your target group and plan activities which take account of their experience and knowledge. For example, if your target group is mainly literacy teachers, a session dealing with maths anxiety is advisable. When working with teachers with little background in numeracy, you may find that sessions take longer than anticipated. Remember that it is important to take the time to build confidence and skills; you do not want to confuse or alienate participants by rushing through the program!

What to include?

Decide on the overall aims of the workshops. It could be a general introduction to teaching numeracy or have more specific goals such as:

- changing attitudes to maths (e.g. overcoming anxiety, building confidence);
- exploring innovative teaching methods (e.g. hands-on materials, cooperative learning);
- focusing on specific content. (e.g. metrics, fractions, problem solving).

At this point the kit should be read carefully and appropriate activities selected for your workshop.

When?

Decide on a workshop timetable that suits participants. Factors to take into account are the distances people have to travel, the money available, time of year and so on.

Possible alternatives are:

- a series of half-day workshops with one or two weeks between them. These are highly recommended because participants have more time to absorb and reflect upon the content of the workshop. They also have the opportunity to try out ideas with their students and share the outcomes of this experience at the next session.
- a series of evening workshops, at weekly or fortnightly intervals.
- whole day workshops - either one, two or three consecutive days.

3. Preliminary Organisation

Publicity

Contact your target group several weeks in advance, to allow time for them to plan their schedules, mail their application and receive confirmation. Make sure your brochures or fliers are attractive, friendly and welcoming, preferably in a distinctive colour so that they stand out from other paperwork teachers and tutors receive. If you use coloured paper make sure it can be clearly photocopied if people want to further disseminate the information. That means using bright but light colours. One example of a publicity flier is shown on page 16. The checklist on page 14 will help you ensure that the necessary information is included.

Registrations

Have an exercise book or list handy to the phone to record registrations and take down the inevitable telephone registrations. For phone registrations, make sure you take a contact number in case of venue or time changes.

When the maximum number has registered, start a waiting list, asking them to phone **you** back a few days before the workshop to see if there are any cancellations.

Always book in more people than you actually want because inevitably some registrants will fail to appear on the day due to family problems or work crises. We suggest overbooking by about 15 - 20%.

How many participants?

Workshops can be planned for any number of people, from six to sixty, ideally using more presenters as the size increases. Do not organise a large group until you are an experienced presenter. Large groups can still operate in workshop format, but there is less opportunity for discussion, feedback and interaction with individual participants.

Using small groups

Small working groups are the key to successful workshops. All the activities in the kit are designed so that participants work in pairs or small groups of four to six. This maximises the opportunity for discussion and sharing of ideas and experiences. It is also a model for cooperative learning, a teaching strategy we strongly advocate for teachers to use with their own students.

Venue

Choose a venue which is easily accessible, close to public transport or parking. A venue with a pleasant atmosphere helps create a positive attitude. We have found that the availability of natural light is an important consideration for whole day workshops. Inspect the venue before making a booking, if possible. Since you are planning the workshop for small groups to work together it is essential to have moveable furniture and suitably sized tables. Use the *Checklist for Venue* on page 14 to help you ensure that the venue is appropriate.

4. Structuring the Workshop

When planning your workshop keep the following points in mind:

Use a variety of activities

Make sure there is a balance between listening, talking and doing. This will ensure that participants remain involved and interested.

Choose activities people can use

Select some activities which participants can use or adapt with their own students. Participants will feel that their time at the workshop has been worthwhile if they can take away concrete ideas and immediately apply what they have learned.

Plan your schedule

When planning your workshop make sure you allot time for registration, breaks and evaluation. It is helpful to write start and finish times for each activity on your own copy of the daily program.

There needs to be some space in the agenda to build in participants' needs that may not have been planned for in the day's activities. If the program extends for longer than one day, these needs can be accomodated in later workshops. There may also be particular local issues which arise, and may also need time for discussion.

Keep to schedule

Once you have planned the workshop, try to keep to schedule. Start and finish on time, and allow enough time during breaks for informal networking, discussion and social contact. During sessions, it is your job to keep people on task, and if necessary, round up discussions so that you can progress to the next activity. Try to keep to plan, but be prepared to sacrifice an activity if valuable learning or discussion is taking place and a particular activity runs longer than planned. Rushing people who are not confident themselves with mathematics can increase their personal maths anxiety and their anxiety about teaching numeracy and defeat the purpose of the workshop.

Use checklists

When planning activities make note of the materials and equipment needed. This becomes a checklist for what to take on the day. The list of basic essentials is provided on the page 15 for you to photocopy and use for each workshop you run.

5. How to Start your Workshop

Get there early to prepare the room, organise the tea and coffee, turn on heating or fans and deal with any unanticipated problems. Arrange chairs around tables in groups of four to six.

Have tea and coffee ready when people arrive. Welcome participants and give out nametags.

The moving start

A constructive way to start the workshop is to make use of the 'moving start' which provides participants with activities to engage in as they arrive. This could include a display of books, equipment and games or more structured activities such as problem solving 'stations'. (These are described fully in Activity 4.4 *Problem Solving Stations*.)

Welcome

When it is time to commence the day's program, ask participants to be seated. **Introduce yourselves,** giving a little information about your own background, experience and interests and talk briefly about the intended aims and outcomes of the workshop.

Outline the day's schedule, including times for breaks and the finishing time. Mention any other important information such as location of toilets and smoking regulations.

Now **introduce participants.** In a small group, everyone could say who they are, where they are from and perhaps give one reason why they are attending or one thing they hope to gain from the workshop. In a larger group this process can be time consuming, so we suggest that participants share this information with the others at their table.

The Activities

The first activity of the day should serve as an icebreaker or warm up, so that participants are interacting with each other from the start.

An excellent way to start a workshop with a large number of participants and set the scene for the activities which follow is Activity 2.1 *Startling Statements*. It gets people moving about and mixing and generates discussion of a range of issues in numeracy learning and teaching.

Other activities which are also useful icebreakers are Activity 3.1 *Sharing Experiences of Maths* and Activity 3.2 *How Adults Learn*. The first of these activities provides participants with an opportunity to get to know each other by comparing their experiences of learning mathematics, and discussing their feelings about numeracy teaching. In *How Adults Learn*, participants share their experiences of learning as an adult (both good and bad) and develop principles for good practice in the teaching of adult numeracy.

6. Evaluating Your Program

How do you establish whether or not you have met the goals of your program? It is important for the staff development team to meet and reflect on the success or otherwise of the program, both at the end of each day's activities and on the completion the program. It is also essential to obtain feedback from participants. Both of these aspects of evaluation are discussed below.

Reflecting on the program

Discussion of the program could include reflection on issues such as organisation, content and subjective impressions of how things went. Some examples are provided below.

Organisation

- Was the venue suitable?
- Did we time things appropriately? (e.g. activities, meal and tea breaks, start and finish times)
- Was the publicity appropriate?
- Was the catering adequate ?

Feelings and impressions

- What was the feeling in the group? (e.g. cooperative, resistant, friendly, tense, relaxed)
- How well did we, the workshop leaders, work together?
- How did we, the workshop leaders, feel at the end?

Activities

- Which activities were received well? Not so well?
- What changes can we make to improve activities?

- Which activities should be deleted? (An important question when you realise that the time available does not allow for your original selection.)
- What further or alternative activities are needed?

From this discussion you can form a list of recommendations which will assist you to plan the next session or staff development activity.

Participant feedback

We recommend that you allocate five to ten minutes each day for collecting feedback from participants. Ongoing evaluation enables you to make changes to the program as you proceed, and ensures that the program continues to be relevant and useful to your target group. At the conclusion of the program the feedback you receive from participants will help you evaluate the success of the entire activity, identify unmet needs and plan future workshops. Evaluation also provides useful information for reporting to funding bodies. Participant feedback is a valuable source of information for supporting requests for funding for future staff development activities. A final reason for evaluaton is that if you have given a good workshop, the positive feedback makes the hard work and subsequent exhaustion worthwhile!

How to evaluate?

There are a number of models available for obtaining feedback, each with its own strengths and weaknesses. These are outlined below, and examples are given in the following pages, to assist in the task of designing a feedback form. Select an approach (or combination of approaches) which you feel is appropriate.

Once the information is collected, it should be read, discussed and acted upon. Compile a list of recommendations for future workshop sessions while the workshop is still fresh in your mind. The feedback can also be collated and incorporated into your final report.

a. Unstructured Response Cards

A simple but effective method of evaluation is to give each participant a sheet of blank paper or a card (index cards work well) at the conclusion of the workshop and ask for comments. This approach gives people the freedom to express themselves spontaneously without restriction, but you may not always obtain views about the issues you are particularly concerned with.

This process can be structured a little by asking participants for responses to particular questions, such as:

- What did you find relevant and useful?
- What would you like more of next time?
- What other topics would you like included?
- Any comments on organisation, childcare, venue or catering ?

These questions can be used verbally or clearly displayed on a sheet of butchers paper or on a whiteboard.

b. Open Response Items

If there are specific questions you want answered, these can be printed on an evaluation sheet either as sentences to complete, or as questions to be answered with room provided for participants' responses (see Example 1 on page 18). Make sure you also allow space for general comments.

c. Numerical Scales

You may wish to evaluate the activity using a numerical scale, such as a five point scale from 1 (poor) to 5 (excellent). This is a useful way to get an 'average' rating from each activity conducted. Activities which obtain a low average rating would then need to be significantly revised or dropped from future workshops. We recommend that you also leave space for general comments. An example of this approach is shown on page 19.

d. 'Before and After' Model

This approach to evaluation focuses on participants' expectations and the extent to which they were met. It enables them to express what their expectations were at the start (which helps workshop planners judge the effectiveness of publicity and workshop descriptions) and comment on how well they were met. (See Example 3 on page 20.)

A final note

When collating your evaluation data, keep the following points in mind.

- How many participants filled them in?
- What was the overall response?
- What were the more important individual comments?

Finally, reflect on whether participants not only enjoyed themselves but also the extent to which they :

- made links to their prior knowledge;
- gained a better understanding of mathematics;
- developed the confidence to use with their own students the methods and materials they experienced in the workshop.

Checklist for Publicity Flier

- Title of Workshop
- Description and/or summary of content
- Names of Leaders
- Target Group
- Time, date and place - a map or street directory reference may be helpful
- Register-by date clearly visible with details of how to register
- Tearoff slip - for prospective participants to return to you - including participant's name, address, phone number and information about their needs and background
- Child care - specify whether available and how to request it
- Payment - if available, needs to be stated giving clear directions on the process, such as the forms participants need to fill in
- Cost
- Catering details (e.g. whether meals are provided)
- Contact person - address and phone number

Checklist for Venue

- Furniture -you will need moveable tables and chairs
- Tea and coffee facilities
- Whiteboard or space for butchers paper
- Adequate light, ventilation, heating
- Pleasant atmosphere
- Appropriate size for the group
- Wheelchair accessible
- Adequate parking and/or accessibility by public transport

Checklist of Basic Essentials

- Tea (including herbal)
- Coffee (including decaffeinated)
- Sugar
- Milk
- Biscuits (Cheese and biscuits as well as sweet biscuits)
- Cups & spoons
- Whiteboard markers and felt pens
- Butcher's paper
- Blu-Tac or sticky tape
- Name tags
- Blank paper
- Evaluation forms
- Books and equipment for display

Handouts:

Equipment:

NUMBERS COUNT IN ADULT LITERACY

A series of three half day workshops for literacy teachers and tutors who would like to incorporate some numeracy in their adult literacy classes and don't know where to start.

> *To be fully literate in today's world our students need not only to be able to read and understand information given in words but also information which appears in numeric and graphical form. Making sense of this information and having the skills and confidence to solve everyday problems involving numbers is what numeracy is all about.*

Many literacy teachers lack confidence in this area and do not realise that they already have most of these skills. All they need are ideas and resources for bringing these concepts into their literacy classes.

The workshop will explore a range of teaching issues and content areas including:

- **The relationship between literacy and numeracy**
- **Cooperative learning**
- **Using the metric system**
- **Games and other group activities**
- **Using a calculator**
- **What is numeracy? What should we teach?**
- **Logical thinking and problem solving**
- **Teaching basic operations**
- **Overcoming anxiety and avoidance of numbers**

Workshop Details

Presenters: Jane Connell and Judy Pappas

Cost: There is no charge for this activity.

Place: Richmond Library: 415 Church Street, Richmond
(*Insert Street Directory Reference, map number and grid reference.*)

Time:
Friday May 4 9.30 a.m. - 12.30 p.m.
Friday May 18 9.30 a.m. - 12.30 p.m.
Friday June 1 9.30 a.m. - 12.30 p.m.

Register by: Friday April 27

Enquiries: Judy Pappas 423 1750

This series is being organised in conjunction with regional Basic Education officers from the Metropolitan Region. Payment for sessional teachers is available through regional staff development funds. Phone your local Basic Education officer for an application form.

NUMBERS COUNT IN ADULT LITERACY

If you are interested in attending the series of workshops complete the form below and send to:

Judy Pappas
Metropolitan College of TAFE
High Street
PRESTON 3072

I am interested in attending the series of workshops to be held on May 4, May 18, and June 1.

Name: ....

Address for correspondence:

....

....

....

Phone: (work)..................... (home)......................

Current Teaching Involvement:

....

....

Place of work: .

....

....

Please indicate your main reason for attending the workshop:

....

....

....

Example 1: Open Response Items

NUMBERS COUNT IN ADULT LITERACY

Session no: Date:

WORKSHOP EVALUATION

General comments and feedback:

Which of the activities presented would you use with your students?

What further needs do you have, or what topics would you like to cover in future workshops?

Example 2: Numerical Scales

NUMERACY WORKSHOP EVALUATION: DAY ONE

1. Please evaluate each of the following activities by circling the appropriate numbers.

	Activity	Poor		Average		Excellent	Comments
1.	Startling Statements	1	2	3	4	5	
2.	Experiencing Success with Metric Length	1	2	3	4	5	
3.	Maths on the Supermarket Shelves	1	2	3	4	5	
4.	How Close Can You Get?	1	2	3	4	5	
5.	Digititis	1	2	3	4	5	
6.	Multidigit	1	2	3	4	5	
7.	Problem Solving Stations	1	2	3	4	5	
8.	Sharing Experiences of Maths	1	2	3	4	5	
9.	Experiencing Fractions	1	2	3	4	5	
10.	Cooperative Logic	1	2	3	4	5	

2. Please tick the activities you would use or adapt for use with your students.
3. What further needs do you have, or what topics would you like to cover in future workshops?

4. Further comments and suggestions.

Example 3: Before and After Model

NUMERACY WORKSHOP EVALUATION

Before the Workshop:

What do you expect to gain from this workshop?

....

....

....

....

After the Workshop:

My expectations were/were not met because:

....

....

....

....

In future workshops I would like more:

....

....

....

....

In future workshops I would like less:

....

....

....

....

Chapter 2. What Do We Teach?

This chapter explores the basic mathematics skills needed to function effectively in daily life, at home and at work. The chapter is divided into six sections, each of which begins with detailed background notes and includes descriptions of the activities within it. The sections are:

- What is Numeracy? 22
- Measurement and the Metric System 40
- Space and Shape 59
- Number Skills 76
- Calculators and Adult Numeracy 86
- Problem Solving. 96

What is Numeracy?

Often when adult students come to class seeking mathematical help they are given a diagnostic arithmetic test which reveals problems with some basic skills such as subtraction and division, and of course, fractions and decimals.

From there the teacher devises a scheme to redress those deficiencies in their basic skills. After some time is spent going over a method, say, for subtraction, the student is furnished with worksheets in which many subtraction sums are laid out on the page, and off they go, lots of sums, lots of practice, lots of ticks. The students' expectations of maths classes have been met, they work happily for two hours a week, and feel they are making progress.

But are they really? Are they developing the numeracy skills which will help them in situations outside the classroom ? Is this all they should be doing? Is this what numeracy is all about ?

There are two aspects to consider in answering these questions.

1. The **definition of numeracy** and what it entails. Does numeracy only imply number skills?
2. In fact, what **number skills are really used by adults** out of school situations?

In her article *Numeracy and Society: the Shifting Ground* Sue Willis discusses these questions, saying very little attention has been given in Australia to investigating the mathematical needs of adults at home or at work. We must therefore rely on research done in England and Wales by the Cockcroft Committee who commissioned studies which sought to find answers to these questions.

The analysis of this research led to the document known as *The Cockcroft Report,* officially *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools.* One of the studies carried out is called *Use of Mathematics by Adults in Daily Life.* It asserts that:

The mathematical needs of adult life include:

- *The ability to record numbers and to count, to tell the time, to pay for purchases and to give change, to weigh and to measure, to understand straightforward timetables and simple graphs and charts and to carry out the necessary calculations associated with these* (Paragraph 32).
- *The feeling for number which permits sensible estimation and approximation and which enables straightforward mental calculations to be accomplished* (Paragraph 33).
- *Sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed whether this be little or much* (Paragraph 37).

In our own investigations during 1989, through discussion with adult basic education teachers and tutors, we identified the following mathematical needs of adult basic education students.

1. Empowerment and Survival Skills

Money: change, basic percentages. Having the confidence to ask appropriate questions about banking, investment, etc.

Measurement : metric system. Being able to visualise and think in metric units and convert within the metric system of length, mass, volume, time.

Space and Shape: recognising signs and symbols, using a street directory and maps.

2. Attitudes and Feelings about Maths

Overcoming anxiety and avoidance of numbers. Providing opportunities for students to recognise the maths skills they already have and use in their daily lives. Building students' confidence in their ability to learn and use mathematical concepts.

3. Number Skills

Understanding of numbers: relative size and value, and place value.

Number Facts : multiplication tables and number facts for addition.

Basic operations: of addition, subtraction, multiplication and some division.

Estimation: predicting results of calculations and checking whether they make sense.

Decimals: in the context of money and measurement.

Simple fractions: understanding and using commonly encountered fractions.

4. Calculators

Using a calculator for solving practical problems, checking results of calculations and for developing number and problem solving skills.

5. Statistics

Understanding simple graphs and statistical information.

6. Logical Thinking and Problem Solving

Deciding which operation is appropriate in practical situations and real-life contexts. Applying problem solving strategies in unfamiliar situations.

7. Maths Language and Vocabulary

Understanding the meaning of mathematical signs and symbols (of mathematical and everyday variety): being able to express numbers, mathematical ideas and concepts in everyday language.

Translating real situations into appropriate mathematical language: for example, 'How much taller than Joe is Sylvia?' becomes a subtraction problem.

Looking at the two lists above we see that local findings and British reports are in agreement that being **numerate** or **mathematically literate** involves much more than number skills and paper and pencil arithmetic. We must ensure that these are not the entire content of numeracy classes.

References

Cockcroft, W.H, (1982) *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr. W.H. Cockcroft.* Her Majesty's Stationery Office, London

Willis, S. 'Numeracy and Society: the Shifting Ground', in Willis, S.(ed.1990) *Being Numerate: What Counts?* A.C.E.R Hawthorn, Vic.

The Activities

This first section provides activities designed to raise participants' awareness of some issues related to the teaching of numeracy to adults.

Activity 2.1 ***Startling Statements*** is an activity based means of raising some of the important issues discussed above and other ideas and statistics relating to literacy/numeracy teaching. It is an excellent icebreaking session with which to open a conference or day long workshop. It is especially suitable for larger groups of 20 or more participants.

Activity 2.2 ***Maths Diary*** invites participants to reflect on their own use of maths related skills in the last few days. This brainstorming of routine tasks makes teachers and tutors aware of how many of their own skills are taken for granted, what knowledge and skills should be encouraged in students, and in particular the importance of estimation in our daily lives.

Activity 2.1 Startling Statements

Rationale

This activity owes its origins to the work of EQUALS at the Lawrence Hall of Science, University of California, Berkeley. EQUALS is a teacher education program which has devised innovative teaching methods designed to encourage girls and minority groups to continue with mathematics.

This activity is an ideal way to start the day's program because it encourages participants to interact, and thus functions as an icebreaker as well as a means of raising important issues about adult numeracy.

Aims

Apart from its role as an icebreaker, the aims of this activity are to raise participants' awareness of:

- the extent of numeracy difficulties in Australia;
- gender segmentation in the Australian workforce;
- important numeracy skills such as estimation and metric measurement;
- non standard methods of calculation;
- an innovative activity which can be adapted and extended in a numeracy class;
- the media as a useful source of information for numeracy activities.

Materials and Equipment

- Overhead projector (optional)
- Calculators
- Masking tape

Time 45 minutes

Handouts

- 2.1A *Some Startling Statements* (optional)
- 2.1B *Answer and Instruction Sheet* (one for each participant)
- 2.1C *Group Instruction Sheet* (one for each question used)

Preparation

Photocopy and cut out the *Startling Statements,* so that each participant has one question. For example, with 20 participants, make two copies of each question or select five questions and make four copies of each. (We recommend that you locate local information and incorporate one or two questions which have particular relevance for your group, if you have time to do this.)

Prepare OHP transparencies of the list of questions or photocopy them for distribution and easy reference in the discussion.

Make one copy of Handout 2.1B *Answer and Instruction Sheet* for each participant.

Make ten copies of Handout 2.1C *Group Instruction Sheet*, one for each question. For a large group (more than 30) these can be posted around the periphery of the room, clearly labelled with the numbers from 1 to 10. Participants with the same question will then move to these 'stations' to discuss the results. For a smaller group one instruction sheet simply needs to be handed out to each pair or small group which has the same question.

Conducting the Activity

1. Briefly introduce the activity. (Refer to the Rationale and Aims of the session.)
2. Attach a question to each participants' back, making sure they do not see it. For a larger group, assistance from participants greatly reduces the time this takes.
3. Give each participant a copy of Handout 2.1B *Answer and Instruction Sheet.*
4. Ask participants to circulate, and collect five answers to the question on their back. Remind them to make sure they do not answer the same question twice.
5. When this is completed, refer participants to the remaining instructions on the sheet.
6. When participants have formed these groups, direct them to the appropriate 'station' or ask groups to do it together. Give each group one copy of the group instruction sheet.
7. Allow five to ten minutes for groups to work out their averages, discuss results and answer the questions on their sheet.
8. Ask participants to move to seats for the whole group discussion. (If you have decided to distribute copies of all ten questions to each participant, now is a good time to do it.)

Discussion

Taking each question in turn display the question on the overhead projector (or read it out). Ask a spokesperson from the appropriate group to report back and record the average, highest and lowest responses. Then reveal the correct information. Some of the estimates may be very close to the correct answers, others will be **startlingly** different!

As you go, discuss each question, referring to the information provided in the following pages, and the issues raised by the groups.

Background Information on the 1989 National Survey of Australian Adult Literacy

Items 1 to 5 come from the results of the report of the first *National Study of Adult Literacy* (Wickert 1989). This survey used a sample of 1500 adults to identify the incidence and and nature of literacy difficulties in the Australian population. Three forms of literacy were investigated, **document, prose** and **quantitative** literacy.

Document literacy was assessed using information located in documents such as a driver's licence, map, phone book, medicine label and a bank deposit slip. Prose literacy was assessed using tasks in which readers were required to locate, produce, interpret and organise information from textual material, such as newspaper articles. Quantitative literacy was assessed using tasks which required the application of numerical operations to information contained in print material, such as a menu, deposit slip, a financial transaction record and a flight schedule.

Reference

Wickert, R.(1989), *No Single Measure: A Survey of Australian Adult Literacy.* Commonwealth Department of Employment, Education and Training, Canberra.

(Available from the Faculty of Education, University of Technology, P.O Box 123 Broadway NSW 2007)

Some Startling Statements

1. In the first national survey of adult literacy in Australia (1989), how many of the 24 items involved numeracy skills?

2. According to the 1989 National Adult Literacy Survey, what percentage of Australian adults cannot calculate their change from $5 for a lunch bill?

3. According to the 1989 National Adult Literacy survey, what percentage of Australian adults cannot accurately work out a 10% surcharge on a lunch bill?

4. According to the 1989 National Adult Literacy survey, what percentage of Australian adults are unable to identify the correct children's dose of Panadol tablets?

5. In the 1989 National Adult Literacy survey, what percentage of the sample were able to correctly locate an intersection in a street directory?

6. 63.5% of Australia's teachers are women. What percentage of the country's engineers and building professionals (e.g. architects and surveyors) are women?

7. How many different ways might be used to find an answer to 29 + 28 =?

8. How many glasses of wine are there in a four litre wine cask?

9. How much does the average Victorian household spend each year on food and non-alcoholic drinks?

10. How many people in this room own a pair of Reeboks?

Startling Statements: Answer and Instruction Sheet

Responses to my question:

1.

2.

3.

4.

5.

Rough average:

Instructions

1. Obtain 5 different responses in answer to your question.

2. When you have 5 responses take the question from your back. Find others who have the same question as you do, or move to the appropriate 'station'.

3. Work out the average of your 5 answers. This can be a rough estimate, a calculated average, or simply the middle number. (You may want to get others to help you with this.)

4. Form a group with the other people who have your question and follow the instructions given.

Group Instruction Sheet

Each group please decide and record:

1. **Lowest** of all the collected responses:

2. **Highest** of all the collected responses:

3. A rough estimate of the overall **average** of your averages:

4. Consider your final answer. How likely do you think it is?

5. What mathematics skills (if any) were involved in finding an answer to the question on your back?

6. What non mathematics (e.g. social) issues are raised by this question?

7. Choose someone to make a brief report.

Answers and Discussion Points for Startling Statements

Question 1

Twenty one of the 24 items in this literacy survey actually included tasks which required the understanding and application of numerical or spatial skills. As well as the six quantitative items, all of the 14 document items and one of the prose items involved numeracy skills. The document items included tasks such as locating an intersection on a map, working out the correct dosage on a medicine label, locating gross pay from a pay slip and writing a cheque. Number recognition was required to identify a swimmer's age from a newspaper article in the prose items.

This question highlights the fact that numeracy skills are embedded in almost everything we do that involves reading and understanding information.

Question 2

38% could not calculate their change from $5.00 for a lunch bill. This item requires people to calculate what change they would receive from $5.00 after ordering two specified items from a lunch menu.

These results highlight the need for strategies to improve the basic numeracy skills of Australian adults.

Question 3

57% of Australian adults cannot work out a 10% surcharge on a lunch bill. The task was to calculate the surcharge on two specified items totalling less than $5. A number of skills were needed to do this task successfully, such as:

- Locating the appropriate information on the menu.
- Understanding the meaning of **surcharge.**
- Knowing how to find 10% of an amount of money.
- Applying this skill in a real life context rather than simply doing a given 'sum'.

Question 4

52% could not identify the correct children's dose of Panadol tablets. (People were presented with a complete packet of tablets for this task.)

Question 5

88% of the sample were able to correctly locate an intersection in a street directory, a more positive result. In level of difficulty, it was similar to circling the expiry date on a licence (85% of the sample) and identifying the swimmer's age from a newspaper article (87% of the sample).

Question 6

4% of Australia's building professionals and engineers are women. (Source: *Women's Work, Women's Pay,* Womens Bureau, Department of Employment, Education and Training, 1990.) This statistic highlights the fact that although women make up 40% of Australia's professional workforce, there is still a high degree of gender segmentation. Women are significantly under represented in scientific and technical occupations, which means that there is still much work to be done in expanding women's career opportunities. Not surprisingly male dominated TAFE and tertiary courses and occupations also require mathematics, either as an integral skill or as a means of selection.

About twice as many boys as girls study advanced mathematics in the final year of secondary school. This limits girls' access to scientific and technical tertiary courses, as these specify mathematics as a prerequisite. It is important to recognise that the key issue here is the **participation rate** of girls and **not** their mathematical ability. Research in Australia and overseas has demonstrated there are no significant gender differences in mathematics ability or performance. For further information on this issue refer to the book by Sue Willis (1989), *Real Girls Don't Do Maths: Gender and the Construction of Privilege*, Deakin University, Geelong, Victoria.

Question 7

This question is intentionally open ended. You may wish to discuss the many approaches to reaching an answer, which include:

1. The standard algorithm i.e.

$$\begin{array}{r} 29 \\ \underline{+28} \\ \underline{57} \end{array}$$

Rounding off and then compensating (in the head methods):

2. 30 and 30 ----> 60 and take 3 away ----> 57

or

3. 29 and 30 ----> 59 and take 2 away ----> 57

or

4. 28 and 30 ----> 58 and take 1 away ----> 57

5. Adding horizontally (a method favoured by school children)

$$29 + 28 \dashrightarrow \underbrace{20 + 20}_{40} + \underbrace{9 + 8}_{17} \dashrightarrow 57$$

6. Counting on (sophisticated method)

$$29 \xrightarrow[+10]{} 39 \xrightarrow[+10]{} 49 \xrightarrow[+8]{} 57 \quad \text{or}$$

$$29 \xrightarrow[+10]{} 39 \xrightarrow[+10]{} 49 \xrightarrow[+1]{} 50 \xrightarrow[+7 \text{ more}]{} 57$$

7. Counting on (basic methods, possibly with fingers)

$$29 + \underbrace{1 + 1 + 1 + 1 \;\ldots\ldots\ldots\ldots\; +1 + 1 + 1}_{28} = 57$$

8. Or even much more basic.....count the lot.

$$\underbrace{1 + 1 + 1 + \ldots\ldots + 1}_{29} \quad + \quad \underbrace{1 + 1 + 1 \;\ldots\ldots\; + 1}_{28} = 57$$

9. Use a calculator (or even a computer).
10. Use an abacus.
11. Use base 10 hands-on materials such as Base 10 blocks, or straws.
12. Ask someone else!
13. Guess.....?

The reason for including this question is to point out the many approaches apart from the standard algorithm for reaching a solution, and that the method depends on the students' skills and the context in which the question is asked. There is no single correct way of reaching an answer in maths and students should be encouraged to explore alternative approaches and choose the one which for them, suits the problem and its context. It is important also to encourage students to practise methods of calculating in the head without always relying on paper and pencil or a calculator.

Question 8

The answer to this question depends on the assumptions you make about how much a glass of wine holds in order to reach a realistic estimate. A standard glass of wine holds about 120 mls, according to the National Alcoholic Beverage Industries Council, but there is no legal requirement for the volume of wine served by the glass in hotels. A glass you fill yourself may hold a little more, perhaps about 160 mls. Using the official standard of 120 mls we should get about 33 glasses from a four litre cask.

This question highlights the numeracy skills of estimation, metrics, and using a calculator if necessary to make calculations. In many situations a reasonable estimate is the most sensible answer. It is therefore important to teach estimation skills to our students

Question 9

The 'average' Victorian household spends $5252 per year on food and non alcoholic drinks. (*The Age*, 16th Oct.1990, reporting ABS figures.)

This question builds on our own knowledge base to estimate a reasonable answer, including our household spending, our concept of 'average', our knowledge of the number of weeks in a year, how we multiply by 50 or 52 and so on.

There are many numeracy skills embedded in apparently simple questions.These can also be extended in a classroom situation to develop students' understanding of statistical concepts such as 'average'.

This question reminds us of the continuous stream of information in the media which can be used in a basic maths class. Students can be encouraged to find their own 'Startling Statements', and present questions to the class. *Startling Statements* can also provide a means of raising awareness about social and political issues.

Question 10

This question is intended to raise a laugh or two. It often turns out that teachers may have bought Reeboks for their children but can't afford them for themselves! Of course this question is not intended for numeracy students, for whom more appropriate questions should be used.

Check estimates from a show of hands.

This format also lends itself to estimating and collecting fairly simple information which is readily accessible from your students. Trivial information can provide a light hearted touch to a lesson. Select topics that would be of interest to students, such as: 'How many Collingwood supporters do you think we have?'; 'How many people own a dog?'; 'How tall is the average student?' You can encourage students to make up their own questions, collect responses and find out the answer through counting or measuring. The activity can also be extended into a graphing activity, by asking students to draw a picture or a diagram of their results.

Activity 2.2 Maths Diary

Rationale

This activity is designed so that the group of participants reflect on their own everyday activities, decide what mathematical skills they use, and by sharing ideas, compile a list of mathematical skills they consider are necessary to function as an independent adult in our society.

The activity also highlights the fact that mathematics involves more than mere number skills: once participants begin to analyse the mathematics involved in the simplest tasks in their lives, they realise how much it pervades our culture, how many spatial, numerical and logical skills they actually use themselves and the disadvantages of not having these skills. It also emphasises the role estimation plays in the calculations done outside the classroom.

Aims

- To examine the mathematical skills involved in everyday life.
- To decide which of these skills are priorities for students.
- To recognise the importance of estimation skills in most aspects of our daily lives.

Materials and Equipment

- Butchers' paper

Time 30 to 45 minutes

Handouts

- 2.2A *Maths Diary*
- *What is Numeracy?*(Background Notes)

Preparation

Read the background notes to this section (p.22) and background notes: *What Number Skills are Important*? (p.76). A related session is Activity 2.6 *Real Maths School Maths: a Discussion of the Video* (p.80).

Conducting the Activity

Divide participants into groups of four to six. Give out **one copy** of Handout 2.2A to each group. Go briefly over the examples given.

Ask each group to discuss some of their activities during the last few days which involved some mathematics or mathematical concepts. Ask them to fill these in on their group's sheet. Allow about 15 minutes for discussion before you ask for a **brief** report back from each group.

Ask one person from each group to describe the tasks and the skills they decided were embedded in each task.

Record these on butchers' paper in columns as shown below.

It is usually necessary to summarise what the reporter is saying, for example the reporter may say something like:

> *The task is getting to an appointment on time. The skills involved are estimating how long it will take to get ready, how long it will take to travel there, reading the timetable to decide which bus to catch ....*

Your summary will probably be

TASKS	SKILLS
getting to appointment	time estimation
	reading timetable
	telling time

When the list is completed, with every group having contributed at least one task, it is time to examine the information collected. To do this, allow participants a few minutes to look over the findings of the whole group and reflect on them. Helpful questions might be:

- *In looking at these skills, are there any 'surprises'?*
- *Are there any skills which appear more often than expected?*
- *Are there any skills which appear less often than expected?*

Discussion should focus on the common elements, i.e. skills which occur over and over.

In particular it should become clear that **estimation** is one of the most commonly mentioned words: we estimate amounts of money, distances, time and quantities. From these estimates we make many of the numerous daily decisions in our lives: what we can afford to buy, the tasks and activities we can fit into the day, how far we can drive before refuelling, how long it will take to reach our destination and so on.

Select some of the tasks listed and ask participants:

> *What strategies would you use to work out something like this?*

Common Arithmetical Shortcuts

Some direct questions which you may wish to ask are:

- *How often do you actually use pencil and paper arithmetic in these tasks?*
- *What do you do instead?*

Answers may include mental arithmetic, calculators, rounding off, sketching diagrams, measurement, visual estimation and many more.

Make sure you elicit as many of these strategies as you can from the participants.

In particular, if it is not covered elsewhere in your program, a few minutes could be devoted to the issue of mental arithmetic as opposed to pencil and paper algorithms.

You could make a list of some of the basic short cuts we take for granted, e.g.:

- rounding off to 'estimate' in calculations;
- adding zeros when multiplying by 10 or 100;
- moving decimal points when multiplying or dividing in money calculations;
- and many others which will undoubtedly be suggested by the group.

Having discussed the everyday skills the group members themselves use, it is time to think about how this relates to school maths from their own experience and what is being taught in their own programs.

Leading questions to ask:

- *How do these skills relate to what you were taught at school?*
- *How do they relate to what you are teaching or planning to teach your students?*

Round Up

Finally give to each participant a copy of background notes *What is Numeracy?*. Ask them to look over the list of skills briefly and compare the information with the topics listed on the butchers paper.

Final Questions

- *Do these recommendations of what is important to teach school children differ widely from what we discussed today?*
- *Have we omitted any major areas?*
- *Do you think they are important?*

Extension (for teachers of workplace mathematics or bridging courses)

Participants could discuss how to find out what mathematical skills are actually used in the workplaces which are relevant to the future careers of their students.

Maths Diary

As a group, think about your activities over the last few days, and record any which had maths content, using the format shown below.

Task	Maths Skill
Deciding whether to go to the bank before I go out tonight.	Estimation, addition, subtraction
Making a pair of shorts.	Metric measurement, fitting shapes together, using parallel lines, imagining a three dimensional object from a two dimensional pattern.

Measurement and the Metric System

The ability to measure, record and perform calculations to do with time, length, weight, volume and area are basic skills for a numerate person. For numeracy students, becoming confident with measurement is one means of feeling more comfortable in the world. Measurement related calculations also provide us with an excellent real life context in which to practise number skills. There are many aspects of measurement upon which we should concentrate with our students. These are discussed in the following pages.

The Use of Measuring Instruments

We often take for granted that adults can read clocks, thermometers, tape measures and all the other such instruments that make up the paraphernalia of modern life. In fact many adults do not feel at all comfortable with measuring equipment and tend to avoid using much of it. For adults brought up with the Imperial (British) system of measurement the change to metric units has caused particular difficulties, some inconvenient and some even dangerous: metric weights in recipes when the user has no idea how to interpret them; parents and children unable to communicate about quantities because they work in totally different systems; and the instances of parents overdosing children with drugs because dosage instructions on the label are in millilitres and mean nothing to them.

Ensuring that adult students can use common metric measuring instruments should be part of any numeracy course. This includes instruments such as:

- clocks - including digital, those with hands, and the 24 hour clock;
- thermometers - including medical, weather, and oven thermometers;
- weighing scales - especially bathroom and kitchen scales;
- volumetric measuring devices - including kitchen measuring jugs, spoons and medicine glasses;
- rulers and tape measures - including dressmakers' and builders' tapes and other specialist devices used in students' employment, hobbies or further studies.

Other instruments of general interest to students might be:

- direction finding compasses;
- protractors;
- micrometers or feeler gauges;
- trundle wheels.

Plenty of practice should be provided at using the instruments in practical situations.

Everyday Use of Metric Units

Alongside teaching the use of measuring instruments we need to teach an **understanding** of the metric units and the **conventions** associated with them.

For instance when teaching about length measurement and the use of tape measures and rulers it is common to begin with students measuring in centimetres. At the same time it is important to discuss the **contexts** in which centimetre measurements are used and the abbreviations used to express them. Centimetres are used in clothing manufacturing, body measurements in passports and at health centres; in fact most measurements to do with bodies and clothes are given in centimetres, and women tend to become more familiar with these.

In contrast, in hardware shops and the world of building, a traditionally male dominated domain, centimetres are rarely used. In these trades, they tend to talk and to draw plans in millimetres. This practice can be very confusing to someone who has carefully measured up their requirements with a centimetre tape and is not prepared for the sudden switch to the large numbers which occur when the measurements are translated into millimetres.

Such experiences can be extremely discouraging and daunting for a student who has just gained some new confidence with the metric system. It is helpful if they are forewarned and given practice at converting to and from millimetres and reading in millimetres directly from a tape or ruler.

Other measurement conventions useful to look at are those for cooking, such as the relationship between millilitres, litres, cups and spoons. Also the units used in packaging and labelling in supermarkets are interesting to examine, for example the connection between half a kilogram and 500 grams. For some students, learning the conversion of grams to decimal fractions of a kilogram is appropriate as it is so often seen on packaging and electronic scales, thus 450 g = .45 kg.

Another area of interest might include the meaning of cc (cubic centimetres) used to describe engine capacity[1]. It is also interesting and useful to know that the cubic centimetre measurement of capacity is exactly equivalent to a millilitre.

Understanding the **connections between units** is made easier once the language of the metric system is examined.

Language of the Metric System

Often students do not realise the connections between the units in the metric system. The meaning of the prefixes **centi,** and **milli** which should make the system meaningful are lost on many adults (not only numeracy students) who have never had the opportunity to learn about or discuss the new system. Many people will buy 600 mls of milk without ever realising that mls is an abbreviation of millilitre or that it is in any way connected to a litre measure.

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[1] A complete explanation of this is given in *Mathematics a New Beginning* (p.MM26) See References.

A short time discussing the meaning of prefixes is always rewarding in an adult numeracy class. An effective means to begin involves drawing on students' current knowledge with the prefix **centi** by brainstorming familiar words in which it occurs and their meanings, such as:

cent:	100 **cents** in a dollar
century:	100 years in a **century**
	100 runs in a **century** in cricket
centipede:	100 legs on a **centipede**

Once this use of the prefix is understood the use of **milli,** as a **thousandth** part (1/1000) is easier to comprehend, and the words **millimetre, millilitre** and **milligram** all suddenly make sense.

Likewise the recurrence of the prefix **kilo** in **kilometre, kilogram, kilolitre, kilojoule**, and **kilowatt** can be made meaningful.

We can even shed light on the modern use of **k's** in computer jargon (e.g. 64 k meaning 64,000 characters of information can be stored in the computer). And finally, the latest unfortunate use of **k** in job advertisements, e.g. salary: $80 **k per annum** meaning a salary of $80,000 dollars per year. Students will usually come up with many examples of their own once you encourage them to look at the words in this way.

Other aspects of language associated with measurement that should be addressed in numeracy courses are the conceptual words associated with the properties or attributes being measured. Words such as long or short, heavier or lighter, width, depth, and height, more than or less than, all should be given attention, especially for students who have learning disabilities or, for whom English is a second language.

Some of these students, may also need assistance in identifying exactly what is meant by measurements of chest, waist, back length and the like, used when buying clothes.

Thinking Metric

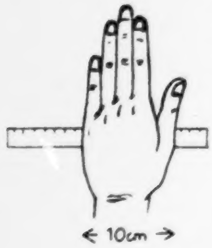
For students who have grown up with the Imperial system one of the most important things to concentrate on is the ability to **think metric**: the ability to estimate distances, lengths, heights, capacity, area and weight using metric units.

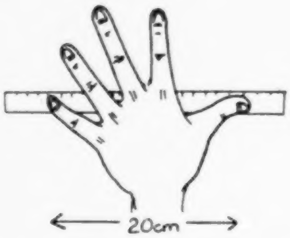
It is better to encourage students to **think** in metric units than to be continually **converting** from one system to another.

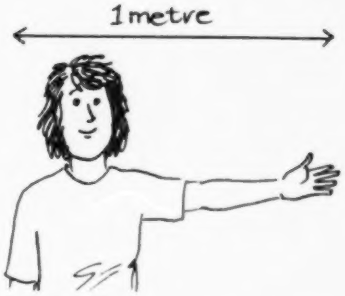
Mostly this ability involves experience and practice. It is therefore up to the numeracy teacher to begin the thinking and assist students with techniques and strategies which will empower them to use the metric system with confidence.

One of the most useful strategies concerns using your own body as a reference against which to measure other things, for example knowing your own height in centimetres is a beginning for making sense of other peoples' heights; thinking, are they shorter or taller than I am? Similarly, knowing your own weight in kilograms is a beginning to interpreting facts about the weight of other people.

We can also use our hands as guides to shorter lengths such as 10 cm and 20 cm. and our outstretched arm as a guide to the metre.







Similarly, known package volumes such as a **1 litre carton of milk** or a **2 litre orange juice** packet, with which we might be familiar, help us to judge visually other unknown volumes.

Familiar items such as a **60 gram egg**, a **375 gram pack of margarine** or a **one kilogram packet** of washing powder may give us some benchmark or reference with which to compare weights.

By assisting students to recognise those aspects of the metric system with which they are **already familiar** and to use these as references by which to judge the unfamiliar, we can begin to demystify the metric system and build a new confidence in our students. Providing positive experiences in class encourages students to practise estimating on their own and gradually become as conversant with the metric system as many of us were with the old system.

To facilitate thinking metric it is a good idea to precede all **measuring** activities by **estimating** or **guessing**. This not only provides more practice at estimation but sets up an expectation that we do have a rough idea of what the measurement will be. It can also provide motivation for the students to be interested in what the actual measurements are.

References

There are many books which contain useful ideas for teaching about measurement and the metric system. Some of those designed specifically for adults are:

Marr, B. and Helme, S. (1988) *Mathematics a New Beginning*. State Training Board of Victoria.

Goddard, R. Martin, J. Marr, B. (1991) *Strength in Numbers*. Eastern Metropolitan Council of Further Education, and Division of Further Education, Victoria. (Both available from The Teaching Maths to Women Project, C/O Northern Metropolitan College of TAFE, St. Georges Rd Preston Victoria 3072)

Norrish, D. and Lord, S. *Undoing some Maths Knots- Numeracy Materials for Women*. Adult Basic Education, Petersham College of TAFE, 27 Crystal St. Petersham NSW 2049.

The Activities

Activity 2.3A ***Metric Estimation*** is designed so that participants can recognise and become aware of their prior knowledge of the metric system whilst at the same time concentrating on the estimating and problem solving skills needed by their students. The activity begins with a metric estimation quiz, **Handout 2.3A** ***Mind Over Metrics***, containing a wide variety of questions to be considered in pairs or groups. Some of the questions require general knowledge, and others a more imaginative problem solving approach. The quiz is followed by discussion of knowledge and strategies used and provides teachers with a starting point to devise some estimation activities which will be useful in their classes.

If insufficient time is available for this activity one of the following **Metric Activities** can be used to begin discussion and focus on participants' prior knowledge.

Metric Activities **(Handout 2.3B)** are three short, stand alone, metric general knowledge activities. They are accompanied by discussion notes and information handouts They are designed to focus discussion on many aspects of the metric system and to allow participants and students to become more aware of their existing knowledge; either prior knowledge, or new facts and skills learned during the workshop.

These activities can be used singly, in conjunction with other types of metric experiences such as estimation and measuring activities, or merely given as handouts for participants to take away and use with their numeracy students. The three activities are of varying levels of difficulty:

Metric Mix 'n Match is suitable for people with very little confidence and is probably more suited as a take away activity for students than for teacher tutor participants.

True or False is a simple exercise looking at some aspects of the general knowledge we expect of adults. It is an effective exercise with either participants or numeracy students.

The One Most Likely requires a mixture of general knowledge and the ability to visualise roughly in metric units. Although the alternatives presented are not difficult, it is the most demanding of these three activities and can act as an effective discussion starter in a metric workshop for teachers and tutors.

Answers and Discussion Points for Metric Activities contains some facts and figures that may be needed by teachers, tutors and staff development presenters when discussing solutions to the above activities. If the activities are used in the workshop then these discussion notes should be given to participants only after the activities have been completed in the workshop.

2.3C ***The Metric System*** is a simple two page handout to distribute to participants and numeracy students so they can clarify and remember some of the fundamental facts of the metric system.

Activity 2.3 Metric Estimation

Rationale

Part of the role of numeracy teachers and tutors is to assist students to **recognise** their **existing** knowledge of the metric system. Once prior knowledge is established and discussed it provides a comfortable basis for calculating and estimating confidently within the metric system.

The skills and calculations involved in estimation do not necessarily come naturally to students in numeracy classes and need to be developed through example and practice.

In this activity we use a metric estimation quiz to focus the attention of the participants on their own current knowledge of the metric system, and on the skills involved in estimation.

Aims

To increase attention participants' awareness of:

- their own working knowledge of metric units used in everyday situations;
- skills used in problem solving situations involving estimation;
- the knowledge of the metric system necessary to function adequately in domestic and workforce situations.

To develop participants' own skills in metric estimation.

Materials and Equipment

- Thermometer (optional)
- Calculators (optional)

Time 45 minutes

Handouts

- 2.3A *Mind Over Metrics*
- 2.3B *Metric Activities*
- 2.3C *The Metric System*

Preparation

Photocopy Handouts 2.3A, 2.3B and 2.3C one for each participant (but only hand out one for each pair during the activity).

Conducting the Activity

Try to have the participants form pairs with people who share similar teaching experiences. A literacy teacher paired with a maths teacher may feel intimidated by the maths teacher's greater familiarity with the strategies and metric units involved. Maths teachers together may get more out of the activity.

Hand out *Mind Over Metrics*, one to each pair, and allow ten minutes to complete the quiz.

Make a comment of the kind:

> *I don't expect you to* **know** *the answers to the quiz, but use any knowledge you have to come up with a reasonable 'guestimate'. Talk about each question carefully with your partner and make sure both of you know how your answer is obtained. No rulers or other* ***conventional*** *measuring instruments are to be used at this stage.*

Discussion

When time is up, go through the questions with the entire group.

Distribute the extra copies at this stage, so that all participants have a copy to refer to and keep.

You may need to elect a volunteer recorder for this to run smoothly.

For each problem on the handout collect a range of numerical answers.

Also collect information on strategies, skills and prior knowledge used to obtain these answers. Some of the questions do have correct answers. These are supplied for your assistance at the end of this activity.

Questions you could ask:

- *What previous experience or knowledge was required?* (e.g. knowledge of specific metric units, visualisation of previously known lengths, the capacity of a bucket.)
- *What assumptions did you make?* (e.g. the size of an average egg, or a carton of milk.)
- *What strategies did you use?*
- *Which strategy was the most effective / imaginative / original?*
- *What calculation skills were used?* (e.g. division, multiplication, addition, subtraction, making comparisons.)

Make a list of these answers to some of the questions as they are mentioned.

Discuss how the answers could be checked or calculated exactly by a group of numeracy students, and what skills could be taught, practised or reinforced by them doing this.

When the list of strategies, knowledge and skills has been made for individual questions, ask participants to look at it and see if any are mentioned often. Make a final list of these **commonly used** skills for participants to keep in mind when working with their students.

Discuss whether participants felt more comfortable tackling this task in pairs than if they had been working alone.

Round Up

Finally discuss with the group which, if any, of these questions they think they could use with their students.

Ask them to devise some questions which would be feasible and act as a worthwhile starter or motivator to measuring and calculating activities. Stress that students will be more interested in measuring and calculating if they have a stake in knowing the outcome. If they have had a guess first, and the activity is begun with questions such as : 'What do you think ... ?' or 'How many ... do you think ..... ?', then students are more likely to get involved in the ensuing activity than if you just issue a sheet of investigative questions. Questions such as these can be done one or two at a time with numeracy students.

Some Facts to Assist with *Mind Over Metrics* Discussion

The **distance from Brisbane to Cairns** by air is 1 392 km.

You could compare this with the similar distances:

Brisbane to Melbourne	1 376 km
Brisbane to Adelaide	1 617 km

Air distances from Brisbane to some other major cities are:

Brisbane to Sydney	747 km
Brisbane to Darwin	2 852 km
Brisbane to Perth	3 611 km

The **weights of eggs** in Victorian supermarkets are:

middies	49 g
standard	55 g
large	61 g
jumbo	67 g.

1 **litre of water** weighs exactly 1 kilogram.

A **bucket** holds roughly 10 litres.

Handout 23A

Mind over Metrics

An activity for **teachers** and **tutors**

Working in your pair or small group try to produce reasonable estimates to these questions using **only metric units.**

- How far is it from the front door of the building to this room ?
- How much does an egg weigh ?
- What is the temperature in the room today ?
- How far is it from Brisbane to Cairns ?
- How many cups of white tea can you make with a carton of milk ?
- How long is a match ?
- How thick is a match ?
- How high is the ceiling ?
- What is the weight of a bucket of water ?
- If everyone in the room lay head to toe, how far would we stretch ?
- How long would it take to wash and dry two dozen glasses ?

Metric Mix 'n Match

Mount this page on cardboard. Cut out all the rectangles, and ask students to match each long rectangle with the most suitable short rectangle.

The height of a household doorway	**50 metres**
The width of an adult hand	**3.5 kilograms**
The weight of a newborn baby	**10 centimetres**
The length of an Olympic swimming pool	**$5\frac{1}{2}$ grams**
The weight of one litre of water	**2 metres**
The width of a little finger	**300 litres**
The weight of a medium egg	**250 millilitres**
The capacity of a medium fridge	**55 grams**
The weight of a ten cent piece	**1 centimetre**
The volume of a kitchen cup	**1 kilogram**

Metric Activities

True or False

Read the statements below. Put **T** in the space if you think the statement is probably true or an **F** in the space if you think the statement is probably false.

[ ] Denis bought a 2 kg chicken to cook for dinner.

[ ] After work Soula walked 24 km to the local shop to buy bread and milk.

[ ] A womens' dress size 12 will fit a bust measurement of 160 cm.

[ ] Jasmine cooks soup in a big pot that holds 8 litres.

[ ] Neville called the doctor because his baby's temperature was 39.5°C.

[ ] A half a litre of milk is the same as 500 ml of milk.

[ ] The boiling point of water is 32° Celsius.

[ ] Water freezes at 0°C.

[ ] Swimming 1000 metres is harder than swimming 1 kilometre.

[ ] The thermostat on most air conditioners is set at approximately 14°C.

[ ] A household bucket holds about 2 litres of water.

[ ] 400 grams of flour is the same as $\frac{1}{4}$ kg.

Metric Activities

The One Most Likely

Circle the most likely answer for each sentence.

1. Karina is 105 centimetres tall. Is she likely to be:
   - a baby
   - a child
   - an adult ?

2. Suzanne has 12 litres of water. Is she likely to:
   - drink it
   - wash with it
   - swim in it ?

3. Wafia measures 10 grams of sugar. Is she likely to:
   - make jam with it
   - use it in a cake
   - stir it into coffee ?

4. Max travels 400 metres. Is he likely to be:
   - driving to the country
   - walking to the local shop
   - flying overseas ?

5. Jim's favourite plant is now fully grown. It is 30 centimetres high. Is it likely to be:
   - a flower
   - a shrub
   - a tree ?

Metric Activities

6. Georgia is $3\frac{1}{2}$ metres above the ground. Is she likely to be:
   - fixing the roof
   - flying in a plane
   - standing on a freeway overpass ?

7. Sammi pours out 10 millilitres of liquid. Is it likely to be:
   - a bucket of water
   - a milk drink
   - a dose of medicine ?

8. Jo sets the temperature at 180 degrees Celsius. Is it likely to be on:
   - a deep freeze
   - a room thermostat
   - an oven ?

9. A body of water is 20 metres across. Is it likely to be:
   - a river
   - an irrigation channel
   - a lake ?

10. The temperature outside is 0° Celsius. Would you be most likely to wear:
    - a T shirt
    - a windcheater or sweatshirt
    - an overcoat ?

The Metric System

The metric system was devised in the Eighteenth Century by the French, at a time of great political and social upheaval known as the French Revolution. As an outdated political system gave way to a new one, so did a new system of measurement replace the old.

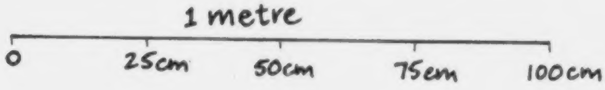
Australia adopted the metric system in 1970.

Unlike the old Imperial units, **metric units** are totally interconnected and logical.

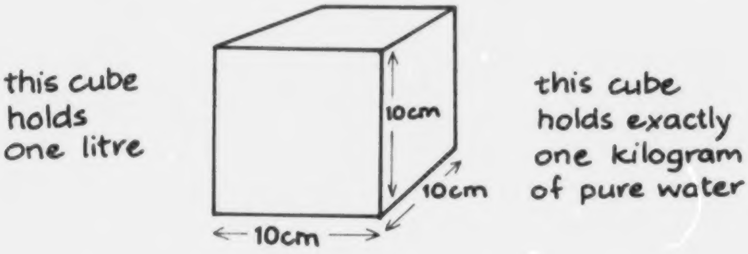
They were devised as follows:

The **metre** was defined as one ten millionth (1/10,000,000) of the distance from the equator to the North Pole (through Paris, of course!)

The metre was then divided into **100 parts**, which were called **centimetres**.


1 metre
0
25cm
50cm
75cm
100cm

The **litre** was then defined as the volume of a cube **10 cm** by **10 cm** by **10 cm.**


this cube holds one litre
10cm
10cm
10cm
this cube holds exactly one kilogram of pure water

The **kilogram** was defined as the weight of **one litre** of pure water.

Handout 2.3C p.2

Metric Prefixes

Kilo means **one thousand** (1000) of

e.g.	**kilo**metre (km)	is one thousand	**metres** (m)
	kilolitre (kl)	is one thousand	**litres** (l)
	kilogram (kg)	is one thousand	**grams** (g)

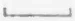
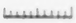
Milli means **one thousandth** ($\frac{1}{1000}$) part of

e.g.	**milli**metre (mm)	is one thousandth part of a	**metre**
	millilitre (ml)	is one thousandth part of a	**litre**
	milligram (mg)	is one thousandth part of a	**gram**

Centi means **one hundredth** ($\frac{1}{100}$) part of

e.g.	**centi**metre (cm)	is one hundredth part of a	**metre**
	cent (c)	is one hundredth part of a	**dollar**

A few helpful facts:

1 **metric cup** holds **250 ml**
1 **level teaspoon** holds **5 ml**
1 **centimetre** looks like this 
There are **10 mm** in **1 cm** 

Answers and Discussion Points for Metric Activities

The three metric activities: *Metric Mix'n Match, True or False,* and *The One Most Likely* are stand alone, metric general knowledge activities. They are designed to act as discussion starters on many aspects of the metric system.

They can be used to help students become aware of their existing knowledge: either prior knowledge; or new facts and skills learned in their numeracy class. They can also be used to highlight areas where more work is needed.

The activities should be used singly over a period of time and in conjunction with other measuring and estimation activities.

The main things you want to achieve with the exercises are discussion, visualisation, estimation and comparisons. Encourage the students to talk about facts and measurements they already know which help them to decide on answers. Only give factual 'answers' when all of their contributions are exhausted.

Metric Mix 'n Match

This activity is suitable for people with very little confidence since nothing needs to be written and all of the correct answers are there for them to find.

Elimination of the easier or measurable quantities is probably the best way to proceed with checking of the answers.

- **The height of a doorway (2 m)** can be checked with a tape measure.
- The **width of a hand (10 cm) and finger (1 cm)** can be checked by measuring with a ruler.
- **The volume of a cup (250 ml)** can be measured with a measuring jug and water.

Members of the class will probably know some of the remaining answers through personal experience in sport, shopping, cooking and the like.The weights could be decided by putting them in order from lightest to heaviest.

- A **ten cent coin** weighs about **5 1/2 grams.**
- Weights of eggs in supermarkets vary from 49 grams (middies) to 67 grams (jumbo). **A medium or standard egg is 55 grams.**
- One **litre of water** weighs exactly **one kilogram.**
- Obviously **baby weights** vary but **3.5 kilograms** is a possible weight.
- Fridge sizes vary but **a medium fridge** is **300 litre.** Ask students to look at the size of their fridge at home.

- An **Olympic pool is 50 metres** long. You could lead discussion to other sports related sizes, e.g. the length of football ovals, racetracks and tennis courts.

True or False

This activity is also suitable for people with little confidence. It covers many aspects of general knowledge which can be elaborated upon depending upon students current level of knowledge and areas of interest.

Some facts which may be helpful for this activity :

Womens' dress sizes

Size	**Bust**	**Waist**
10	80 cm	60 cm
12	85 cm	65 cm
14	90 cm	70 cm
16	95 cm	75 cm
18	100 cm	80 cm
20	105 cm	85 cm

Obviously the statement is false. (You could get students to look for the **number patterns** in this one also.)

- **Cooking pots** or saucepans come in a range of sizes but generally, in a typical set of three saucepans, the smallest is approximately one litre, the medium about 1 1/2 litres and the biggest a bit over 2 litres. (The pot sizes are often also described by the diameters of the base e.g. 14 cm, 16 cm, 18 cm. Larger pots - 2 handle variety - begin at about 3 litres and go up to large stock pots of 19 or so litres. So the statement can easily be true.

  Encourage students to check their own cooking equipment or take some of yours along to measure in class.

- **Average body temperature is 37°C** so a temperature of 39.5°C would be cause for alarm.

  The C stands for Celsius, the name of the person who invented this temperature scale. The important features of this temperature scale are:

  0° is the freezing point of water (so the statement is true.)

  100°C is the boiling point of water. (So 32°C is false for the boiling point.)

- Most **air conditioners** are set at around **20°C.**
- **1000 metres** is exactly **one kilometre.**(So swimming both distances is the same.)
- A **household bucket** holds approximately **10 litres.**

64

- **1/4 kilogram** is 1/4 of 1000 grams which is **250 grams** or .250 kg on the delicatessen scales.
- **400 grams is .400 kg** on the scales. So the statement is false.

The One Most Likely

This is an effective discussion starter. It encourages visualisation and comparison and enables students and participants to share their current knowledge whilst deciding upon answers. It is a good activity for small groups or pairs.

When leading a discussion on this activity participants' knowledge can be further extended if you encourage explanations of peoples' choice of answers. For example in discussing the sugar question you could talk about the other alternatives by asking: *Does anyone know how much sugar is used in a cake? How is it measured ? etc.* This provides an opportunity to share knowledge of spoon measures, cup capacities and the like.

Some answers such as the heights and lengths can be checked by actual measurement or by pacing out of distances, thus providing opportunities for measurement and estimation as well as discussion.

Answers

Question 1

Karina is most likely to be a child. (105 cm comes close to the waist of an adult of average height.)

Question 2

Suzanne is most likely to wash with 12 litres of water. (The capacity of a bucket is about 10 litres.)

A standard cup holds 250 millilitres ($\frac{1}{4}$ of a litre). Ask about the size of a glass.

Question 3

Wafia is most likely to be stirring 10 grams of sugar into a cup of coffee.

10 grams of sugar is the same as 10 mls or 2 teaspoons. (It may be interesting to note that 1 ml of sugar weighs exactly 1 gram - the same weight as water.)

Ask about how much sugar goes into a cake or jam - what volume, what weight, and how it is measured.

Would 10 grams of honey be 2 teaspoons?

Question 4

Max is most likely to be taking a walk if he travels only 400 metres.

You could discuss the numbers of metres in one kilometre (1000), and in 1/2 a kilometre (500).

Question 5

At 30 cm, Jim's favourite plant is most likely to be a flower.

Debate could center on size of miniature shrubs, bonsai trees, and other anomalies. However since lively and interested discussion is the aim of the exercise, debate is to be seen as an advantage rather than a problem as long as it is done using lots of metric language and visualisation.

Question 6

If Georgia is 3 1/2 metres above the ground she is most likely to be fixing the roof. (3 1/2 metres is an average roof height)

You could look around the room and estimate door heights and ceiling heights of the building you are in and compare them to modern or older buildings.

Question 7

Sammi is most likely to be pouring a dose of medicine.

As with Question 3, 10 mls is 2 teaspoons.

Question 8

180 degrees Celsius is most likely to be the temperature of a moderate oven.

Discuss other oven temperatures, likely room temperatures and even the temperatures of freezing water if it arises. (See notes for *True or False*.) This could link immediately to Question 10.

Question 9

At 20 metres across, the body of water is most likely to be a river.

Well, it is to Victorians anyway! However this is another debatable question which may lead to interesting discussions of river, canal and lake sizes both in Australia and overseas. It could be helpful to pace or measure an exact 20 metres so people can get a picture of it. Comparison with an Olympic pool might be useful.

Question 10

If the temperature outside is 0°C you are most likely to be wearing an overcoat (or all three garments).

Discussion of local compared to overseas minimum temperatures could follow this one.

Space and Shape

These days numeracy means more than numbers, and includes a range of mathematical concepts and skills used to perform many tasks in everyday life. In this section we focus on the skills which people use to operate in the three dimensional world we live in.

Thousands of activities in daily life - cutting up vegetables, constructing a garment or a piece of furniture, paving a patio, ordering concrete, folding a letter, crossing the road, driving a car - require us to respond to spatial information, or make use of spatial concepts, such as distance, area, speed, angles, and relationships between moving and stationary objects.

The skills embedded in these activities we tend to take for granted. But numeracy students may not have had the opportunity to develop these skills to the best of their ability. For instance, they may not know how to:

- read warning symbols and logos (e.g. symbols indicating dangerous chemicals);
- locate an intersection in a street directory;
- give or follow directions (e.g. 'Take the second turn off on the left, on the north side of town');
- visualise a 45° angle to comply with parking signs;
- estimate distances on a map.

The following section provides more details of the spatial skills which are most relevant for adult numeracy teaching.

What spatial skills are important for numeracy students?

In the 1991, a national statement on mathematics for Australian schools was produced. The result of a collaborative project by mathematics educators in all states of Australia, it described the mathematical understandings, skills, knowledge and processes which should be made available to students and which should enable them to meet the mathematical demands of everyday life.

Space is one of the eight strands of mathematics identified by this statement as a fundamental aspect of mathematics education. The document notes that:

> *We use spatial ideas for a wide variety of practical tasks. We describe our surroundings, find our way around, and mark out and construct living spaces. Spatial ideas are basic to the solution of many design problems: producing inexpensive but sturdy packaging, laying out a page for maximum appeal, arranging pattern pieces to minimise waste or developing the orbital engine* (p.78).

The project also recognised the importance of **visualisation** in developing spatial concepts and skills. *Mathematics in our Schools*, a guide for parents and the community which accompanies the statement, comments that:

> *Well developed visualisation skills will improve a student's capacity to follow directions and identify shapes, arrangements and locations in different orientations. They will also assist a student to imagine in advance what a shape or arrangement will look like. As well as being a powerful tool in the study of mathematics generally, the ability to visualise is directly applicable to a wide range of occupations, such as dress design, architecture, engineering, town planning, carpentry and sculpture* (p. 26).

What does this mean for the adult numeracy student? Below we have outlined in some detail the skills and concepts which are important for adult numeracy students. This list is intended as a guide only, as the skills and concepts covered will depend on the skills, knowledge and needs of your students.

1. Shapes and their properties

Students need to be able to identify the geometric shapes which commonly occur in our environment and which we need in order to negotiate our environment safely. This would include:

- Recognition and naming of plane figures, for example triangles, circles, rectangles, squares and hexagons; and solids such as cubes, spheres and cones.
- Understanding of angles; being able to describe common angles and understand how they relate to each other, for example that 90° is a right angle (or quarter turn).
- Confidence in using the concepts and language of distance, direction and shape, including length, width, area, diagonal, diameter and radius. (E.g.What towns are within a 20 km radius of Albury?)
- An awareness of symmetry as it applies in daily life, for example in laying and cutting out clothing patterns, or describing what shapes are created when a square is folded along an axis of symmetry (e.g. along its diagonal).

2. The relationship between two dimensional and three dimensional shapes

Students need to be able to form meaningful connections between maps, diagrams and drawings and the three dimensional objects they represent.

Constructing shapes and solids helps to develop the ability to visualise three dimensional figures represented by two dimensional plans. These activities also develop skills in following instructions, methods of accurate measurement and spatial knowledge such as an awareness of symmetry.

3. Location and map reading

These skills include:

- Using a map, street directory or atlas, including locating points with a grid reference.

- Giving and receiving directions for locating objects in space (e.g. the butcher diagonally opposite the supermarket).
- Using the points of the compass (e.g. my house is north of the college).
- Using scale to work out distances on a map or street directory.

Useful Resources

The following publications provide a range of spatial problem solving activities which are suitable for numeracy students :

Downie, D. et al. (1981) *Math for Girls and other Problem Solvers.* Lawrence Hall of Science, University of California.

Burns, M. (1975) *The I Hate Mathematics Book.* Cambridge University Press.

Kerr-Stenmark, J. et al. (1986) *Family Math.* Lawrence Hall of Science, University of California.

References

Australian Education Council and Curriculum Corporation (1991) *A National Statement on Mathematics for Australian Schools.* Carlton, Vic.

Australian Education Council and Curriculum Corporation (1991) *Mathematics in our Schools; A Guide for Parents and the Community.* Carlton, Vic.

(Both documents available from the Curriculum Corporation, St Nicholas Place, 141 Rathdowne St Carlton Vic 3053, tel: (03) 639 0699)

The Activities

Activity 2.4 ***Pentominoes*** uses a hands-on approach for exploring geometric concepts and language.

Activity 2.5 ***Creative Use of a Street Directory*** uses cooperative problem solving to focus attention on the knowledge and skills required for using a street directory or map. A range of activities which can be used to develop these skills is also provided.

Handout 2.8B comprises a range of problem solving activities for numeracy students, and includes a number of spatial problems.

Activity 2.4 Pentominoes

Rationale

Cooperative problem solving is a an effective means of developing mathematical skills and understanding, and providing students with the opportunity to describe and explain what they are learning. This activity shows how this approach can be used to explore and develop geometric concepts, skills and language. The particular skill focused on in this activity is the relationship between two dimensional shapes (or nets) and their three-dimensional counterparts.

Aims

The aims of this activity are to:

- Demonstrate the value of cooperative problem solving as an approach to teaching geometric concepts and language.
- Demonstrate links between two-dimensional and three-dimensional shapes.
- Provide participants with an activity which can be adapted or extended for use with their own students.

Material and Equipment

- Scissors (2 pairs per group)
- Whiteboard or butchers paper and pen
- Milk carton with top cut off (optional)

Handouts

- 2.4A *Grid paper* (4 sheets per group)
- 2.4B *The Twelve Pentominoes*

Time 40 minutes

Preparation

Read the introductory notes on space and shape. Photocopy Handout 2.4A *Grid Paper*, so that each a group has four sheets, and one copy of Handout 2.4B *The Twelve Pentominoes* for each participant.

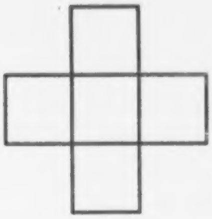
Conducting the Activity

Assign participants to groups of between 3 and 5 seated around tables. Distribute scissors and grid paper (but not Handout 2.4B at this stage). Discuss briefly the importance of geometric concepts, skills and language in everyday life, referring to the introductory notes.

Ask participants to close their eyes and visualise a box without a lid. Ask:

How many sides does it have?

Draw a diagram of the box in two dimensions. (This is also called a **net**). If you have a milk carton, cut off the top, cut down the seams and lay it flat, like so:

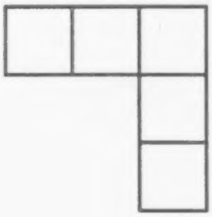


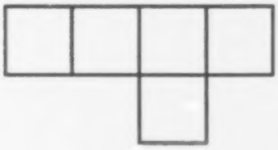
Then tell participants that this shape, made up of **five** squares is called a **Pentomino**. You may like to ask for other words which include the prefix **pent** (e.g. pentagon, pentathlon). You may also wish to discuss in passing the names of other shapes made by combining squares, such as the domino (2 squares), triomino (3 squares) and tetromino (4 squares).

There are two problems to be solved in this activity; the first is to find out how many different pentominoes there are and the second is to work out how many of these actually form boxes when folded up. We suggest that all participants complete the first part of the activity before embarking on the second.

1. How many different Pentominoes are there?

The purpose of this activity is to discover through exploration how many ways five squares can be arranged. Two examples are shown below:





Propose the problem for participants:

> *Your task is to find all the possible ways to arrange 5 squares into pentominoes. Use the grid paper provided to draw your pentominoes. Make sure that all your shapes are really different from each other!*

You will need to explain what makes pentominoes the same (congruent) or different. For example, pentominoes which are mirror images of each other are the **same** because they can be **flipped over** and matched exactly. Also, pentominoes are the same if one can be **rotated** to fit exactly over the other one.

Allow groups 10 to 15 minutes for the task and then discuss the results.

When time is up, ask participants how many **really** different pentominoes they found (there are 12 possible shapes). Discuss any strategies used for making sure that all possibilities were included.

Now distribute Handout 2.4B *The Twelve Pentominoes*, so that participants can check how many they found, and discover ones they may have missed. They will also need the complete set to proceed with the second part of the activity.

2. How many boxes?

Some of the 12 shapes fold up into boxes and others do not. The second part of this activity involves discovering which shapes fold into boxes.

Propose the second problem :

> *Your next task is to sort your pentominoes into shapes which fold up to form a box and those which do not. Start by ticking the shapes on Handout 2.4B which you* ***think*** *fold into a box. Everyone in your group should agree.* ***Then,*** *cut them out to check your prediction. You will need to copy any missing shapes from the handout onto your grid paper first.*

Allow 5 to 10 minutes for the task and then discuss the results:

- *Which ones fold into boxes?*
- *Were your predictions accurate?*
- *Were any of the shapes misleading?*

Discussion

These two activities raise a number of issues about teaching geometric concepts, which can now be discussed. Some questions for discussion could be:

- *What did you learn from this activity?*
- *What were the strengths and weaknesses of working in a group on this activity ?*
- *What geometric concepts and language were covered by this activity?* (E.g. perimeter, area, surface area, volume.)
- *How could you use or modify this activity for basic numeracy students?* (E.g. exploring all possible tetrominoes and investigating whether or not they have the same perimeter.)
- *How could you extend this activity?* Examples are provided below:

  - folding pieces to decide which have mirror symmetry and which do not;

  - sorting pieces by their perimeters. This provides a concrete experience of the notion that shapes with the same area do not necessarily have the same perimeter;

  - making a puzzle with the twelve pentominoes by trying to fit all the pieces into a 12 x 5 grid (yes, it can be done!);

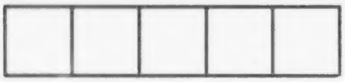
  - proving that there are only 12 pentominoes.
- *What do you think are the main strengths and weaknesses of this activity?*
- *Can you suggest further activities which develop skills in space and shape?*

Grid Paper

Handout 2.4B

The Twelve Pentominoes

1.



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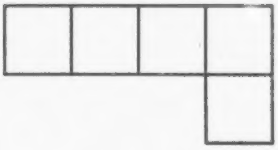

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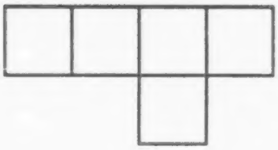

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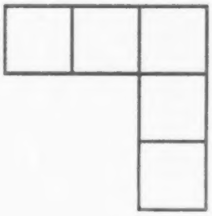

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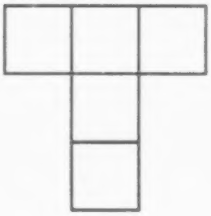

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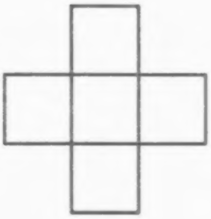

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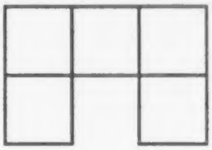

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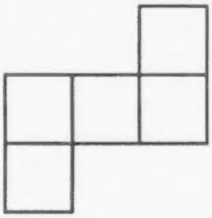

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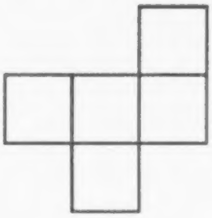

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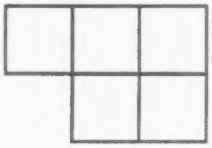

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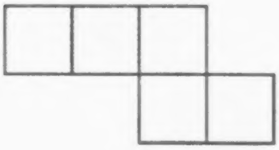
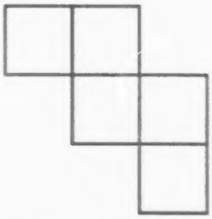


12.



Activity 2.5

Creative Use of the Street Directory

Rationale

In our society street directories are taken for granted, as is the ability to use them. Even those who use them regularly may be unaware of the enormous range of information available in their directories or local city maps.

In this activity two cooperative problem solving tasks are used as a focus for discussion around this topic.

Aims

To raise participants' awareness of:

- the knowledge and skills required for the everyday use of a street directory or map;
- the large range of information available in their local maps or street directories;
- the related skills which are of importance to their own student group;
- a range of activities which can be used to help develop these skills in students.

Materials and Equipment

- 1 set of Handout 2.5A and Handout 2.5B, made for each group of 4-6 participants
- local street directories or maps (as many copies as possible)
- 1 felt pen per group

Time 1 hour

Handouts

- 2.5A *The Stolen Golden Surfing Wagon*
- 2.5B *Where is Grandma's House ?*
- 2.5C *Map of Twin Creeks*
- 2.5D *Street Directory Activities: Ideas for Use with Numeracy Students*

Preparation

Make one copy of Handout 2.5A *The Stolen Golden Surfing Wagon* and Handout 2.5B *Where is Grandma's House?* for each group of 4 to 6 participants. Mount them onto card. The clue cards should be cut up and stored in a labelled envelope. Include at least two copies of Handout 2.5C *Map of Twin Creeks* in each envelope.

Photocopy Handouts 2.5 A to 2.5D, one for each participant.

Read the introductory remarks to Activity 4.6 *Cooperative Logic*.

Conducting the Activity

Divide students into groups of 4 to 6 around suitable sized tables (see discussion in Activity 4.6 *Cooperative Logic* (p.229) or Activity 4.7 *Exploring Group Work with Problem Solving* (p 242).

To each group give out Handout 2.5C *Map of Twin Creeks*.

The first short task is designed to give participants some practice in the use of grid references and to familiarise them a little with the map. Ask the groups to decide what geographic features or public facilities would be found at each of these locations:

F1	[Railway station and bridge over creek]
D4	[Primary School, bus route, telephone box]
C4	[Church, bus route]
E4	[Hotel, bus terminus, parkland]
C5	[Traffic lights, public toilet, police station, bus route]

Now you can proceed with the problem solving tasks. These are essentially cooperative logic activities so if the group has not experienced Cooperative Logic before you will need to explain how it works.

The rules below are identical to those in Activity 4.6 *Cooperative Logic*:

- Each student will be given at least one clue card. All the cards are needed in order to solve the problem, so some group members will receive two cards.
- No one can give their clue cards to another person or lay them down for others to read. They can, however read their card aloud as many times as they, or the group needs, in order to solve the problem together.

Begin with *The Stolen Golden Surfing Wagon*, giving one envelope of clues to each small group. Read aloud to the whole group the story which describes their problem solving task:

> *Donna's old golden coloured surfing wagon was known by everyone in the little town of Twin Creeks. One Sunday morning the town locals were amazed see it racing through the streets at great speed. Donna was usually a careful driver!*
>
> *In fact the car had been taken by a gang from out of town who tried to steal it, but they had to abandon it when they ran out of petrol. On the way they lost a hub cap. With the help of the town locals, Donna worked out the exact route of the stolen car and was able to find the hubcap.*
>
> *Can you draw the route on the map provided and show the exact spot at which the car was found?*

Give the group sufficient time to solve the problem without rushing slower groups.

If some groups are finished and waiting for others, ask them to make a list of the skills and knowledge they used to solve the problem.

Discussing Problem 1

When all the groups have completed the task discuss the question above, about skills and knowledge, with the whole group.

You may find it helpful to make a list as the discussion is taking place.

The main points to be made from this discussion are:

- the meanings of the symbols used on the map and how they can be translated from the key accompanying the map.
- maps are usually presented with North at the top of the page. From this knowledge you can work out the other directions using a variety of 'rules of thumb'.

  One such is N

  W E

  S

  If you face North, South is behind you, and West and East spell out the word WE: West on your left, East on your right.
- how the information in this problem (based on a popular Victorian street directory) relates to directories and maps used in the towns participants and their students are from.
- this activity was designed for tutors and teachers and may be far too hard as a starter for student groups.

Some specific questions to ask :

- *Did anyone learn anything new themselves about map symbols ?*
  [It is common that most people find something new each time they examine a street directory closely.]
- *How did you work out which direction was West ?*
  [Ensure all participants are clear about this before moving on - see above.]
- *Do you think all of this information is available in your local map/directory ?*

Now give out Problem 2.5B *Where is Grandma's House?* Read the story aloud and conduct the cooperative activity as before.

> *Donna's grandmother grew up in one of the oldest houses in Twin Creeks, before many of the houses and roads were built. With the help of these clues, can you find where her house is?*

This problem introduces the use of the **map scale**, giving a **grid reference**, and some of the symbols not used in the first problem. In addition it reinforces directional concepts.

Discussion

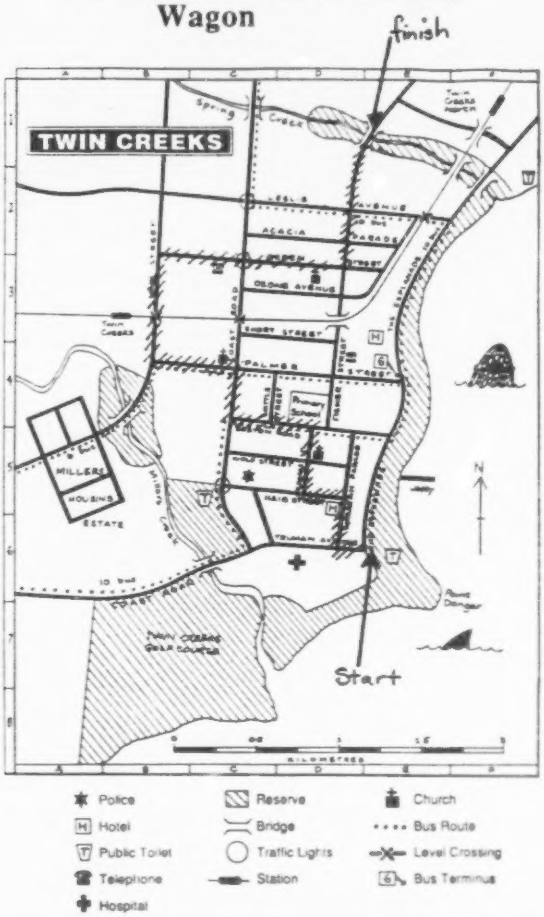
Initial discussion of this problem should focus on the new knowledge required to solve this problem which was not used in the first activity. Then lead in to general discussion of the aspects raised in both problems such as:

- *Which street directory features would be most appropriate to work on with your own students?*
- *Was any useful language introduced in these problems?*
- *Has anyone done any map/directory work already in literacy classes?*
- *What ideas have you used which have worked well?*
- *Could any aspects of these problems be used to introduce mathematical skills or concepts?*

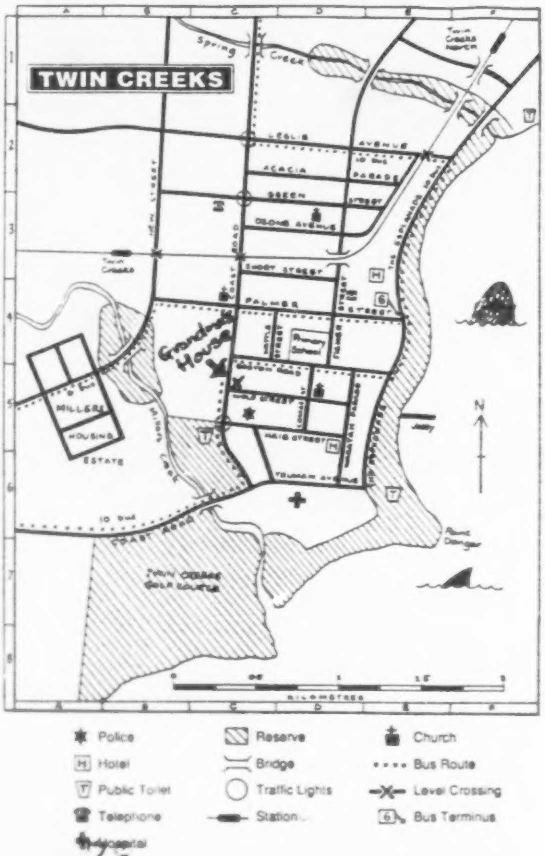
If participants respond to these last questions a list of brainstormed ideas should be made. A list is provided in Handout 2.5D which can be distributed as a supplement to the list generated by the group.

Solutions

The Stolen Golden Surfing Wagon


finish
TWIN CREEKS
Spring Creek
Leslie Avenue
Acacia
Green Street
Ozone Avenue
Short Street
Palmer Street
Primary School
Millers Housing Estate
Coast Road
Twin Creeks Golf Course
Start
Kilometres
Police
Hotel
Public Toilet
Telephone
Hospital
Reserve
Bridge
Traffic Lights
Station
Church
Bus Route
Level Crossing
Bus Terminus

Where is Grandma's House?


TWIN CREEKS
Spring Creek
Leslie Avenue
Acacia
Green Street
Ozone Avenue
Short Street
Palmer Street
Primary School
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Coast Road
Twin Creeks Golf Course
Kilometres
Police
Hotel
Public Toilet
Telephone
Hospital
Reserve
Bridge
Traffic Lights
Station
Church
Bus Route
Level Crossing
Bus Terminus

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Handout 2.5A

The Stolen Golden Surfing Wagon

Problem: to be read aloud to the group before starting.

Donna's old golden coloured surfing wagon was known by everyone in the little town of Twin Creeks. One Sunday morning the town locals were amazed see it racing through the streets at great speed. Donna was usually a careful driver!

In fact the car had been taken by a gang from out of town who tried to steal it, but they had to abandon it when they ran out of petrol. On the way they lost a hub cap. With the help of the town locals, Donna worked out the exact route of the stolen car and was able to find the hubcap.

Can you draw the route on the map provided and show the exact spot at which the car was found?

Clues

The car nearly knocked over old Sam as it turned left at the pub corner.	**The caretaker at the primary school saw the car travelling west past the school.**
Donna had left the car parked near a public toilet on The Esplanade.	**Some kids in a phone box saw it speed straight through a red light beside them.**
The car had to wait at a level crossing as the train pulled into the station.	**As the car sped along Fisher St. it nearly collided with a number 10 bus.**
The car was eventually found beside a bridge in one of the town reserves.	**The car was seen passing all of the town's three churches.**

Where is Grandma's House ?

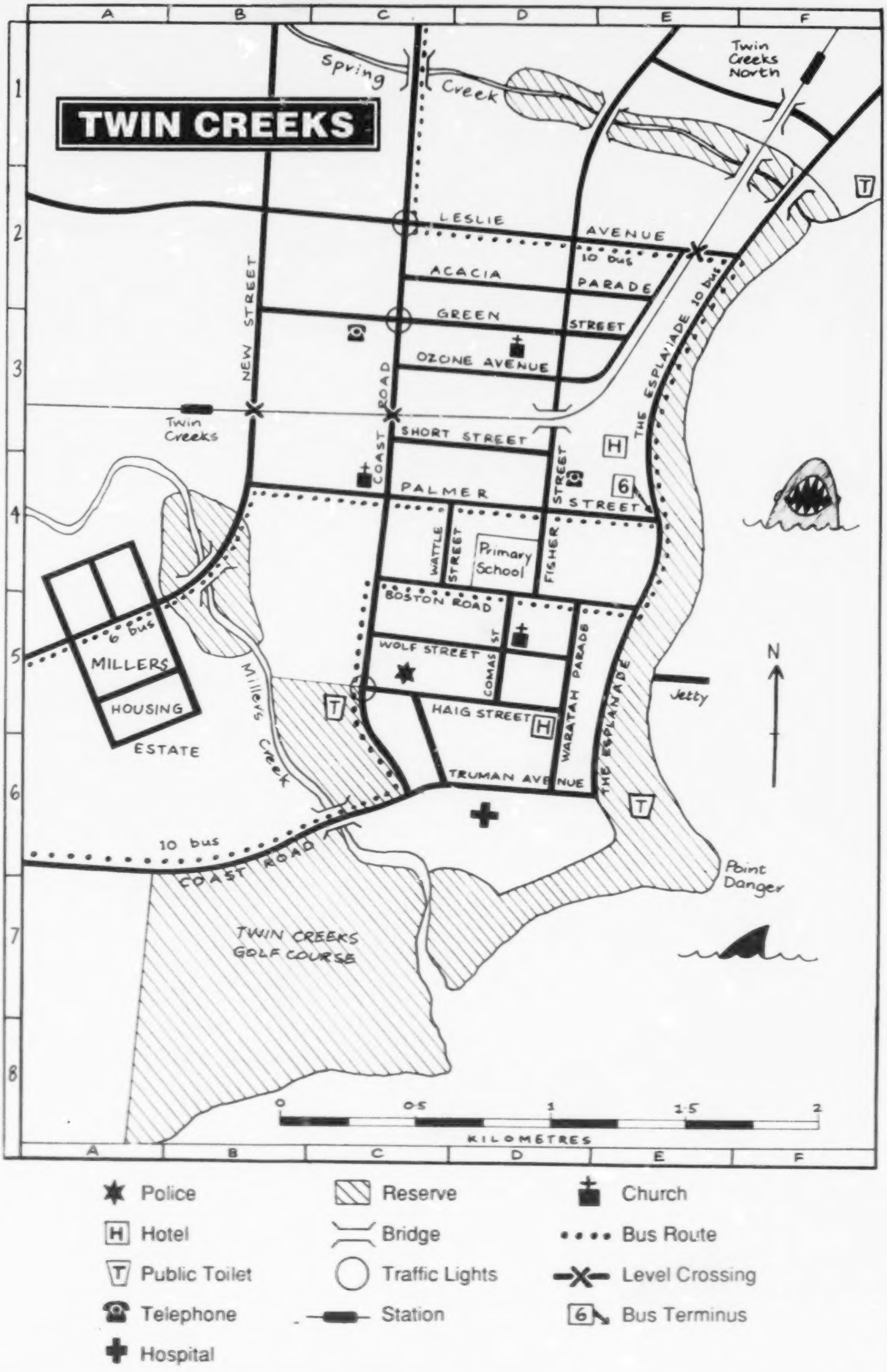
Problem: to be read aloud to the group before starting.

Donna's grandmother grew up in one of the oldest houses in Twin Creeks, before many of the houses and roads were built. With the help of these clues, can you find where her house is ?

Clues

From the front verandah she used to be able to watch the sun set behind Millers Creek.	**The terminus for the number 6 bus is north-east from her house.**
The number 10 bus now runs past her house.	**Her house is on a corner.**
The house is south of Boston Rd.	**Two of the town's churches are equal walking distance from her house.**
Her house is on the north side of the street	**She used to walk exactly one kilometer in a straight line from her house to Twin Creeks Station.**

Map of Twin Creeks


A
B
C
D
E
F
1
2
3
4
5
6
7
8
TWIN CREEKS
Spring Creek
Twin Creeks North
LESLIE AVENUE
10 bus
ACACIA PARADE
GREEN STREET
NEW STREET
OZONE AVENUE
COAST ROAD
Twin Creeks
SHORT STREET
PALMER STREET
FISHER STREET
THE ESPLANADE
WATTLE STREET
Primary School
BOSTON ROAD
WOLF STREET
COMAS ST
WARATAH PARADE
HAIG STREET
TRUMAN AVENUE
6 bus
MILLERS HOUSING ESTATE
Millers Creek
Jetty
N
Point Danger
10 bus
COAST ROAD
TWIN CREEKS GOLF COURSE
0
0.5
1
1.5
2
KILOMETRES
Police
Reserve
Church
Hotel
Bridge
Bus Route
Public Toilet
Traffic Lights
Level Crossing
Telephone
Station
Bus Terminus
Hospital

Street Directory Activities: Ideas for Use with Numeracy Students

- Use indexes and grid references to find:
    - students' own houses
    - location of the class
    - public buildings/facilities of interest or relevance, e.g., town hall, hospital, station.
- Put a large map on the wall (commercial or compiled of photocopies of appropriate street directory pages). Ask students to mark in their homes, the location of the class, and draw on the map the routes by which they come to class. The key map at the front of the directory would also be useful for this.
- Extend either of the above to work out:
    - the distances students travel to class;
    - distances from each others homes, e.g. ask students to find which other student lives closest to them;
    - best routes from one home to another, or to places of interest from their homes, using car, foot or public transport. Such routes could be written down or presented verbally in the appropriate language;
    - how to travel from one point to another by public transport with the least number of changes; (using a Public Transport map is also useful.)
    - how long such a journey would take (in conjunction with timetables).
- Students write a set of instructions for someone to follow to get to:
    - their home
    - a favourite place
    - a useful public building.

  Other students to follow the instructions tracing the route along directory maps and see where they end up. This activity could incorporate a variety of language and skills at levels according to students' capabilities and include:
    - turn left /right
    - travel north/south along
    - follow ... as far as ...

  - make a ten o'clock turn ... turn at the traffic lights or T intersection.

- Students draw a map of their own neighbourhood from memory marking in features of interest using words and symbols (either standard directory symbols or symbols they invent and explain themselves).

- The group plan a 2-5 kilometre walk beginning and ending at the class location, using the street directory scale and symbols. Such an activity could include: what are they likely to pass; how many churches, traffic lights, public buildings, sportsgrounds or other items of interest.

  Go on the walk to check - combine it with a writing exercise.

- Plan a real excursion or outing for the group using maps, timetables, and price charts. Make sure it's somewhere they want to go, and follow it through as an end of course outing, term break, or just a special treat.

Introduction of Mathematical Ideas

If you feel your students are ready to discuss some more formal mathematical ideas, street directory activities can be an easy beginning as described below.

- **Location of points**: two numbers can be used to specify an exact location.
  - In the directory. This is done with A9, D7 etc
  - For coordinate graphs, pairs of numbers such as (3,2), (5,7) locate exact points in relation to a vertical and horizontal axis.

  Note For coordinate graphs the ordering of the number pair is important; the horizontal distance is always specified first. This is the same for some street directories but not those with the letters on the vertical.

- **Angles and Directions**
  - The directions North, South East and West can be described in terms of bearings: the angle turned from the northerly direction in a clockwise direction. East is 90°, South is 180°, West is 270°, and North is 0° or 360°.
  - Types of intersections can be described in terms of categories of angles such as acute (less than 90°) right angle (exactly 90°) and obtuse (between 90° and 180°).

What Number Skills are Most Important?

The Cockcroft Report discusses numeracy as including an **'at homeness' with numbers** and the **confidence** to use them in everyday situations. The report also throws doubt on some of our more traditional beliefs that the majority of people will use the mathematical skills we teach them in school.

> *The methods which are used when at work to carry out calculations are frequently not those which are traditionally taught in the classroom. Employees use a variety of idiosyncratic and 'back of envelope' methods, especially for long multiplication and division. The methods which are used depend very much on the user's confidence in his own mathematical ability.* (Cockcroft 1982, paragraph 71).

There has been a tendency in schools to regard number skills as merely the memorising of multiplication tables and simple addition facts, and the learning of the standard algorithms (step by step methods for performing calculations or mathematical process e.g the steps for long multiplication.) In the past these have been taught with little regard to situations in which they may be used.

Any attempt to teach 'in the head' methods was called 'mental arithmetic' and really was a speed test of remembered facts. Such practices produced an anxiety and nervousness about arithmetic, which always seemed intrinsically bound up with speed and instant recall, rather than inventive and useful number strategies. Instead, mental arithmetic should be be seen as the invention and practice of non standard and helpful methods for doing calculations in our heads, 'the back of envelope' and 'tricks of the trade' techniques so often used by adults outside the classroom.

Calculations Used in the Workplace

It was not from maths teachers that many of us learned quick and useful marketplace arithmetic strategies. It was from adults working in real situations: workers who have devised their own techniques for dealing with numbers in an appropriate way in their work situations. These methods tend to be passed on as part of the job.

For example:

- In pre calculator days, when people bought 5 items for $1.99 each and a quick total was needed, taking out pencil and paper and working out 5 x $1.99 would have been ludicrous. Instead we were taught to round off to $2, multiply the $2 by 5, and then subtract the 5 extra cents to give $9.95.
- When working out the 15% sales tax in the printing business, for instance on $ 138 did we use fractions and write $\frac{15}{100} \times \frac{\$\,138}{1}$?

  No! There was a quick shift of the decimal point, a halving process and a rapid addition.

84

Amount on bill = $138.00
(move decimal point) 10% = $13.80
(halve it) 5% = $6.90
(add) 15% = $20.70

- Again, in printing, when counting out 100 sheets of printed paper for packing we fanned the paper and counted 25 lots of 4 to get our 100 sheets.

The above are but a few of the many examples which could be cited by workers from their own experiences. The variety of methods used in the workplace is endless and depends on the type of figures and calculations relevant to the job. Practices like those described will interest numeracy students who should be encouraged to learn similar techniques and given opportunity to practise them in situations divorced from pen and paper worksheets: quick in the head calculations for shopping totals such as 3 items at $2.99, $4.95 and the like;

- practice at working out 10% and 15% of varying sums of money and devising methods for calculations such as 20% , 12½ % or others relevant to their real life situations; and
- learning and practising facts such as 25 x 4 = 100, 3 x 25 =75, 20 x 5 =100 which can be useful in practical calculations and counting situations and in money and percentage applications.

Apart from work situations, most adults find themselves at some time in their daily routine having to add and subtract in their heads, whether it be for shopping, banking, measuring and building, or merely planning their day in terms of time.

Many people find it more comfortable to do calculations in their heads in what might be referred to as a horizontal manner, as opposed to the more vertical classroom style. For instance when adding 67 + 38 the thinking process might be:

60 and 30 is 90, 7 and 8 is 15 then
90 and 15 is 90 ..... 100 105
and 10 and 5

Similarly, counting down in subtraction is a simple technique used by many people, for instance 83 - 26 might go

83 take 20 83 ... 73 ... 63
now take 6 63 ... 60 ... 57

All these effective methods require experience and understanding of numbers. It is not enough to merely teach algorithms in our numeracy classes. We should also explore ' in the head ' methods and the knowledge and skills underlying them.

What Skills are Important?

All of the methods discussed require an understanding of numbers, not merely an ability to follow the steps of an algorithm. They require real understanding of place value and the composition of numbers, of rounding and estimating and of the meaning of operations and how they relate.

These perhaps are the skills on which we should be concentrating before we rush into teaching algorithms without adequate understanding.

In fact sometimes our students have so much trouble with the understanding of the carrying and decomposing in the algorithms that they continually make mistakes. Some educators are of the opinion that they might be better off without these traditional methods because they lead students away from common sense into doing **an operation** the way the teacher said it should be done.

A vivid picture of this is given in the video *Real Maths School Maths* which shows school children answering a question concerning change when buying a chocolate bar. The calculation was done with ease when it was presented as an 'in the head problem' with real context. The children, however, stumbled badly on the same calculation presented as a formal subtraction on paper.

It may be salutary to read this excerpt from Sue Willis' article *Numeracy and Society:*

> *The boy was just 7 years of age. We were playing a game in which each of us received a score each round. At the end of the first round I wrote down my score and the child's, and repeated this for the second round. Immediately he interrupted me, 'No no you don't do it that way. You've got to do running totals!'*
>
> *'What are running totals?' I asked, and he replied, 'The last time you got 54 and this time you got 27 so you have to put these both together and you just put the running total down each time.' He then proceeded to add my scores for the two games and then his own. From his nodding and his lip movements he appeared to be 'saying' 54, 64, 74, 81. He continued to add the scores mentally (and correctly) with final scores of 500 to 600. I asked how he knew to do that and he replied 'Well I just know it, I just worked it out'.*
>
> *Later that same evening I glanced through some work he had brought home from school and noticed pages of 'add ups' (set out in the traditional vertical form) including a series like '54 +27' which involved carrying from the units to the tens column. Every exercise which did not involve carrying was answered correctly while every one involving carrying was answered incorrectly.*
>
> *A while later I suggested that we pretend to play a game, I would make up the subscores for him to do the running totals. He obviously thought it was a fairly peculiar request. Why would you do mathematics in isolation, if it serves no purpose? Nonetheless, he did it. Of course I chose for my subtotals ones that matched his incorrectly completed written exercises. When he completed them correctly mentally I exclaimed with exaggerated surprise, 'That's funny, how come you got something different here?' and pointed to the written exercise. His reply : 'No', he said , 'They're not running totals, they're add-ups and this is the way you do it. Our teacher told me that this is the way you do it'.(Willis 1990, p.20).*

Many of our adult students have devised rapid techniques of their own for calculations because they have forgotten algorithms for addition or subtraction, or never learned 'their tables'. It is important that we, in our enthusiasm to teach what we may think is 'the right method', do not discount these past techniques. Instead we should reinforce students' prior methods and value them alongside alternative methods that we may provide always remembering that many methods are effective. Even when doing traditional style calculations, students should be encouraged to explore methods of checking their answers either by understanding that subtraction can be checked by adding, and division by multiplication, or by estimation and rounding off techniques.

References

Cockcroft, W.H (1982) *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools, under the Chairmanship of Dr W.H Cockcroft*. Her Majesty's Stationery Office, London.

Willis, S. 'Numeracy and Society: the Shifting Ground', in Willis, S. (ed.1990) *Being Numerate: what Counts?* A.C.E.R Hawthorn, Vic.

Video *Real Maths School Maths* Produced by Bill Newton for the Victorian Ministry of Education. Available from The Education Shop, 117 Bouverie St Carlton Vic. 3053. Tel 03 342 3939. Very likely you can borrow the video from a central Education Curriculum or Resource office in your own state.

The Activity

Activity 2.6 ***Real Maths School Maths: A Discussion of the Video*** is included to motivate discussion of the issues raised above. The session, designed around this very effective video, enables participants to look at the variety of methods adults use to perform one particular calculation and how few of them use formal school methods. It also provides an opportunity for examining the methods of subtraction currently taught in school: how it works; how it compares with the more old fashioned method; and what alternatives we should be presenting to our students. Some of these considerations are also discussed in Activity 2.1 *Startling Statements* (Question 7).

Activity 2.6 *Real Maths School Maths*: a Discussion of the Video

Rationale

This activity uses the video *Real Maths School Maths* to provide an interesting discussion starter to the issue of 'in the head' computations. By looking at a diverse range of methods used by children and adults, when confronted with identical problems, the video allows participants to reflect on the computation practices they are encouraging in their students.

Aims

To provide stimulus for participant discussion and reflection on the following questions:

- paper and pencil algorithms - when do they help, when do they hinder?
- are we providing students with a suitable range of computational skills?
- are we encouraging students to be aware of the variety of methods which can be used to solve any given arithmetic problem?
- what knowledge is needed to use a calculator for calculations with money?

To familiarise participants with the decomposition algorithm for subtraction.

To examine a variety of alternative methods for common arithmetic computations, such as addition, subtraction, multiplication and division.

Materials and Equipment

- Video *Real Maths School Maths* * Produced by Bill Newton for the Victorian Ministry of Education.
- Calculators, 1 for each group

Time 1 hour minimum

* Available from The Education Shop, 117 Bouverie St Carlton Vic. 3053. Tel 03 342 3939. Very likely you can borrow the video from a central Education Curriculum or Resource office in your own state.

Handouts

- 2.6A *Real Maths, School Maths: Preliminary Problems*
- Copies of the article *Real Maths School Maths: Can we Close the Gap?* which accompanies the video (optional).
- 4.3A *Investigating Addition and Subtraction using Straw Models*

Preparation

- Read background notes: *What Number Skills are Most Important* ? (p 76.) You may wish to photocopy these as a handout.

- Watch the video yourself and decide which aspects you wish to concentrate on, and whether you have time for the whole video or only parts of it. The entire video takes only 19 minutes.

  The activity seems more effective if you break viewing into two parts: 1 - the children; 2 - the post office; with discussion immediately after each of them.

- Familiarise yourself with Activity 4.3 *Modelling Place Value, Addition and Subtraction* or an alternative clear way of teaching the decomposition method of subtraction.

Conducting the Activity

Before you show the video it is a good idea to put the problems it examines to the participants themselves. This way they will know their own natural reaction or method of solution before listening to others solving the problems.

Give out Handout 2.6A *Real Maths School Maths: Preliminary Problems*.

Inform participants that getting answers is not important but it is better for them to know their own likely approach before seeing those shown on the video.

You may like to remark:

> *In the video we see the wide variety of methods people used to obtain answers to these questions.*

Have calculators readily available on each table so that participants can use them to reach an answer or as a means of checking.

When participants have had a few minutes to complete these problems, or at least attempt them, show the video.

Stop after the interviews with the children if you are showing it in two parts.

Discussion

Discussion of the video will depend largely on the participant group, their prior knowledge and level of maths skills. If the group is large you may want to break into groups of 4-6 for the discussion. The two parts of the video should be discussed separately.

Part 1: The children

Discussion of this section should cover three main points:

- children **could do the problem well when set in context** but not when it was presented as an **abstract sum** or school maths problem on paper.
- how the borrow and pay back and the decomposition methods are actually supposed to work.
- whether these formal or algorithmic methods are really appropriate for this kind of computation or whether students should be made more conscious of looking for simple strategies.

Some questions to begin this discussion could be:

- *How did the children work out the Cherry Ripe change?*
- *How did you work it out foryourselves?* (Discuss different approaches.)
- *Why did they not use such a simple method when asked to work out*

$$\begin{array}{r}500\\ \underline{-45}\\ \hline\end{array}$$

- *What does this mean for our own teaching?*
- *Which method is faster?*
- *Which method is more accurate?*
- *Could the change method be done on paper? And does it matter if it can't ?*

One way of **writing** could be:

45 ----> 50	----> 100	----> 500
saying 'and 5 is'	'and 50 is'	'and 400 is'
writing (+ 5)	(+ 50)	(+ 400)

the procedure was: 'to get to 500 from 45 you have added 400 and 50 and 5 which is of course 455' (very easy when the numbers are said aloud).

Participants could be encouraged to try this, or any other methods suggested, on a variety of computations to see when it would be effective and when not.

Another approach used by students for the Cherry Ripe change was:

Take 50, that gives 450

then add the extra 5 ... so 455

If this has not emerged in discussion you could give it as an example then expand on the idea with:

- *How often can such an approach be used in working out change?*
- *What about amounts like $10.00 - $5.90 ?*
- *Is it worth trying out such methods with your students?*

Explaining the Decomposition Method of Subtraction

Usually many participants are unfamiliar with the decomposition method of subtraction so you should be prepared to explain it and allow them a chance to practise using it for some easier problems, for example, 34 - 19. Concrete materials, straws or MAB blocks should be on hand to use for explanation and practice (See Activity 4.3A *Modelling Place Value, Addition and Subtraction p.179)*

Part 2: The post office

This section illustrates the variety of methods which can be successfully used to arrive at an answer to one particular problem.

Two interesting points emerge:

- the effectiveness of a guess and check or estimation style of calculation;
- the complexities which arise when a calculator is used to solve the problem.

Guess and check or estimation

Most of the people did some sort of estimation in their heads such as

about 3 for a dollar so 5 x 3

or *Its more than 10, but less than 20...*

Such a technique gave them a starting number to multiply by 30. This was then adjusted as necessary. All of these processes were done quite speedily by those interviewed and none relied on a division process for the entire calculation.

Some of these processes, or alternatives used by your participants are worth discussing. Questions could be:

- *How many of the participants used methods taught them in school to tackle this?*
- *Did the different methods have anything in common?*
- *What role did estimation play in the methods we saw?*
- *Are we assisting our students with these skills?*

The calculator approach

What needs emphasising here is the amount of prior knowledge necessary to use a calculator for this calculation e.g:

- If you enter 5 for $5 then 30 cents must be entered as .3 or .30;
- or $5.00 is entered as 500 cents and 30 cents can remain 30;
- .666 appearing in the answer, 16.666...., represents a fraction of a stamp not an amount of money;
- to calculate the change requires another complete step; most commonly it involves performing the multiplication 16 x 30 and then subtracting from 500.

Possible discussion starters are:

- *A tutor in a staff development activity once said ' If students are allowed to use calculators, why do we need maths teachers? We may as well all pack up and go home.' In the light of this one calculation have you any comments to make?*
- *How many steps does it take to do the calculation on a calculator?*
- *What knowledge and skills are assumed?*
- *Would your students have this knowledge?*
- *How can we assist them to acquire these skills?*

Extension

If you have time to spare use the **17 x 6** problem that Ari was asked in Part 1 of the video.
Starters could be :

- *Remember Ari, the boy who did calculations for* 17 x 6 *in his head so well?*
- *How did he do that calculation?*
- *He ended up by adding 90 and 12. Where did those numbers come from?*

He did something like this

6 x 17

6 is easy but 17 needs to be simplified

think of it as 3 lots of 5 and 2 extra

6 x 5 is easy... it's 30

there's 3 lots of 5 so I need 3 lots of 30, ... 30, 60, 90, ...

now the 2 extra

2 lots of 6 is 12

so the total is 90 plus 12 = 102

- *How many ways could you think of to do this calculation?*

See also Question 7: *How many different ways might be used to find an answer to* 29+28 = ? in Activity 2.1 *Startling Statements* with discussion on p.33)

Round up

Discuss the major points covered in the session and make a written summary. This will depend on your group's focus, but should include:

- standard pen and paper methods are not always the easiest way to do a calculation;
- students apply more common sense when problems are set in real life contexts which they can relate to (for example teaching decimals in the context of money instead of treating them as abstract numbers);
- there are many, many correct ways to get an answer to a given calculation;
- using a calculator is not always as straightforward as we might assume.

Real Maths School Maths

Preliminary Problems

To gain the maximum from the video *Real Maths School Maths*, you should try these problems yourself **before** watching the video.

1. If you buy a Cherry Ripe for 45 cents how much change will you get from $5.00?
(Try to do this as if in the shop: without pencil and paper.)

2. Calculate $\begin{array}{r} 500 \\ \underline{-45} \\ \underline{\quad\quad} \end{array}$ using a pen and paper method.

3. If you go into a post office with $5 how many 30 cent stamps could you buy ?
(Use any method you like with this one.)

4. How much change would be left from the $5 after buying the stamps?

Calculators and Adult Numeracy

Most adults these days, when faced with a computation at work or at home, reach for a calculator. They are a cheap, readily available and reliable tool, so why not teach students how to use them?

Many people do not accept the widespread use of calculators, believing that using them in class is a form of cheating. There is also fear that people become too dependent on calculators and use them mindlessly for calculations which should be done mentally. But extensive research with school age children has demonstrated that using calculators is more likely to **improve** the average student's basic skills. Moreover, students using calculators tend to possess **more positive attitudes** to mathematics than students who do not use calculators (McIntosh, 1990).

Why Teach Calculator Skills?

There are a number of good reasons for teaching calculator skills, and using calculators in the adult numeracy class:

- Using a calculator is an **essential and expected skill in the workplace,** therefore students should be taught this skill.
- Learning to use a calculator **breaks down the fear of modern technology** often experienced by adult students. Once students become familiar and comfortable with calculators, other technology, such as computers, becomes more accessible.
- Used in the classroom, calculators provide **immediate and neutral feedback.** A student can test an answer immediately without embarrassment or loss of self-respect. Errors and misunderstandings can then be rapidly diagnosed and remedied.
- Calculators allow students to work **with real-world data**, which usually involves large numbers which are difficult to work with using pencil and paper methods and standard algorithms. Using current and relevant real-world data rather than contrived data found in standard texts allows students to confront real issues and solve genuine problems.
- Calculators bring computation within the reach of **intellectually disabled** students. Students who are unable to remember basic number facts or the times tables, once taught to use a calculator, are greatly empowered in their everyday activities, such as working out how much money is needed to pay for things or checking their change.

How Should Calculators be Used?

The question for mathematics educators in the 1990's is no longer **whether** calculators should be used along with basic skills instruction, but **how**.

There are numerous examples of calculator activities in a variety of mathematics books.

These are useful for teaching calculator skills, but we urge you to encourage your students to use them alongside **all** numeracy activities, for example to check answers or to engage in real-life calculations. Students who regularly use calculators in this way are more likely to integrate calculators into their everyday lives.

In the adult numeracy classroom, calculators can be used in a number of ways, including the following:

- as a **number-cruncher**, to deal with computations which are complex, tedious or whose performance by pen and paper methods would interfere with the main purpose of the activity. This is very true for low-confidence adult students who feel more secure knowing there is a calculator handy if needed.
- to **check the results of calculations** done using standard algorithms or mental arithmetic.
- to **develop and reinforce basic concepts and skills,** such as operations, place value and multiplication tables. For example, to reinforce the seven times table, students can use the constant function available on some calculators, as follows:

7 $\boxed{+}$ 7 $\boxed{=}$	Answer displayed:	7
(press equals again) $\boxed{=}$	Answer displayed:	14
(and again) $\boxed{=}$	Answer displayed:	21

  Students can predict the next result and check their prediction using the calculator. Many calculator games have now been devised. These are a valuable way of practising basic skills without the tedium of drilled exercises.

- to explore **patterns and relationships** in numbers. For example, using a calculator to investigate and predict what happens when numbers are multiplied and divided by 10, 100, 1000 and so on, students can discover the rules for themselves.
- To enhance students' **understanding of large numbers** by estimating and then calculating. For example, estimating how long it would take for one million seconds to pass. (The result, surprisingly is only eleven and a half days.)
- To encourage **experimentation in problem solving.** When a student has a challenging problem to solve, for example converting $A100 into US dollars and is given the conversion rate of $A1 = 76.4 US cents, they may not know where to start, whether to multiply or divide and by what: 0.764 or 76.4? What they probably **do** know from the information given or from their own experience is that the Australian dollar is worth **less** than the US dollar and that they should therefore get less than $US 100 for their $A 100.

A calculator allows students to **experiment with alternative approaches** until they feel confident that their answer is reasonable. (The answer is $US 76.40.)

Using Calculators Effectively

In order to use calculators sensibly, students need certain skills, many of which were not taught in the mathematics classes of the past.

> *We use technology every day and, as the use of a new technology becomes more widespread, old skills are lost and new skills are developed* (Willis, 1990, p.13).

What skills are important for the effective use of calculators?

- **An understanding of the value of numbers.** For example, to add $1.37 and 13 cents on a calculator requires understanding of the data we are feeding in, in particular, place value. In order to produce the correct result the student has to enter the 13 cents as 0.13, not simply 13. This requires an awareness of what the numbers are worth in this particular context, and how they should be entered on the calculator.
- **Making sense of the calculator answer**. In the previous example, the calculator result is 1.5 because the calculator drops right hand zeroes. This result has to **be interpreted** by the student as equivalent to $1.50 and **not** one dollar and five cents.
- **Estimation skills**. To be aware of whether or not a calculator result is acceptable, students need to have a rough idea of what result they are likely to get. For example, if buying 2.5 metres of fabric at $7.95 a metre students should be looking for an answer between 2 x $8 and 3 x $8, that is between $16 and $24, or about $20. If the calculator answer is significantly different from $20 they know that they have made an error such as pressing the wrong key and need to repeat the calculation.
- **Rounding skills**. The previous example illustrates the importance of **rounding** numbers to an appropriate degree of accuracy, first at the initial approximation (i.e.$7.95 rounded to $8) and second to make sense of the calculator result of multiplying 2.5 by 7.95 (19.875). Rounding up is required in this case, to obtain a final charge of $19.88.
- **Basic number facts**. In the previous example, a knowledge of tables was required to mentally calculate 2 x 8 and 3 x 8. Basic number facts are thus an essential skill for the effective use of calculators. However, students who cannot recall their tables could do the estimates 2 x 8 and 3 x 8 on the calculator.
- **Accuracy in entering data**. It is inevitable that students hit the wrong button occasionally, and thus need strategies for knowing whether or not they have entered the correct data. Such strategies include repeating the calculation until they obtain the same answer twice and/or using their estimation skills to detect unlikely answers and always checking the display after each entry.

- **Selecting the appropriate operation.** Students need to learn which calculation is necessary. Perhaps it is more important these days to know **when** to divide, rather than **how to**.

  For example, to work out how many buses are needed to transport 250 workers to a stopwork meeting when each bus holds 36, the student has to first decide that division is necessary, then enter the numbers in the right order and finally give meaning to the result. The calculator answer of 6.94 does not tell us how many buses; students need to round up to the nearest whole number in order to produce a meaningful answer.

What Sort of Calculators?

For basic numeracy, we recommend a simple calculator with well spaced single-function keys, a positive 'click' action, a percentage key, a memory key and preferably solar powered (so you don't have to worry about replacing the batteries). Large desktop calculators are recommended for students with impaired motor skills.

References

Australian Association of Mathematics Teachers and the Curriculum Development Centre (1988), *A National Statement on the use of Calculators for Mathematics in Australian Schools.*

McIntosh, A. (1990) Becoming Numerate: Developing Number Sense. In Willis, S. (1990), *Being Numerate: What Counts?* Australian Council for Educational Research Ltd, Hawthorn, Vic.

Willis, S. (1990) Numeracy and Society: the Shifting Ground. In Willis, S. (1990), *Being Numerate: What Counts?* Australian Council for Educational Research Ltd, Hawthorn, Vic.

The Activity

Activity 2.7 ***Using Calculators Effectively*** explores the use of calculators in solving a range of numerical problems and highlights the skills and knowledge needed for their effective use.

Activity 2.7 **Using Calculators Effectively**

Rationale

Using calculators is an accepted practice today, at work or at home. In a learning situation, they enable students to solve problems with real world data, which may involve large numbers and cumbersome calculations. This activity explores the use of calculators in solving a range of numerical problems and highlights the skills and knowledge which are needed to use them effectively This activity is also an opportunity for participants who are unfamiliar with calculators to strengthen their confidence and skills and provides a range of question types which can be adapted for use with numeracy students.

Aims

The primary aim of this activity is to increase participants' awareness of the skills needed to use a calculator effectively, which include:

- understanding the value of numbers and the meaning of the decimal point;
- basic number facts (e.g. tables);
- estimation prior to calculating;
- accuracy in entering data;
- selecting the appropriate operation;
- rounding numbers;
- making sense of the calculator answer;
- checking the calculator answer.

Materials and Equipment

- Calculators (preferably non - scientific) one per pair
- Whiteboard or butchers' paper
- Pens

Handouts

- 2.7A *Using Calculators*

Time 1 hour

Preparation

Photocopy Handout 2.7A *Using Calculators*, one for each participant. You will also need to work through the problems on the handout and familiarise yourself with the answers and discussion points.

Conducting the Activity

Referring to the background notes, discuss with participants some of the issues associated with teaching students to use calculators. Questions for discussion could include:

- *Should students have calculators? Why?*
- *How should students use calculators?* (e.g. as a number cruncher; to check results.)
- *What skills and knowledge are needed to use calculators sensibly?*

Introduce the activity, mentioning that it will also provide an opportunity for participants unfamiliar with calculators to get to know them better. Ask participants to form pairs, and distribute one copy of Handout 2.7A *Using Calculators* to each pair.

Read out question 1:

Add $1.37 and 13 cents on your calculator.

After a minute or so, ask pairs to contribute their answers. Do not be surprised if some initial attempts at the problem include $14.37 (by entering 13 instead of .13) and 1.5 (by forgetting to put back the zero the calculator drops off). This example illustrates that using a calculator to solve even a simple problem makes some mathematical demands of the user. Ask the group:

- *What skills are needed to work this problem out using a calculator?* (Answers should include entering decimals correctly on the calculator; interpreting the calculator answer of 1.5 to obtain $1.50)
- *How did you know if your answer was correct or not?*
- *What skills are needed to work out this problem without a calculator?*

Now ask participants to work through the rest of the questions on the handout, making sure that they :

- **estimate** before using a calculator;
- identify the **skills and knowledge needed** to solve the problem using a calculator;
- **check** their answer;
- discuss whether or not the calculator was useful, and if so, how.

Allow 10 to 15 minutes for this activity. As it is inevitable that some pairs will work more quickly, do not expect everyone to complete the questions in the allocated time.

Discussion

Take each question in turn, and referring to the discussion points below, ask participants for answers, and comments on the skills they needed to use the calculator effectively.

Answers and Discussion Points for *Using Calculators*

Question 1 Answer: $1.50
Skills and knowledge: Entering decimals correctly on the calculator; interpreting the calculator answer of 1.5 to obtain $1.50

Question 2 Answer: $19.88
Skills and knowledge: Selecting the appropriate operation; rounding numbers. There will probably be a number of appraoches used in solving this problem; solving the problem with more than one method is a good way to check the solution.

Question 3 Answer: 7 buses
Skills and knowledge: Selecting the appropriate operation; making sense of the answer (you can't have 6.69 buses).

Question 4 Answer: 40 880 litres (about 4 000 buckets).
Skills and knowledge: Basic operations; Number of days in a year; selecting the appropriate operations for solving the problem; rounding to reach appropriate approximation.

You may wish to discuss the use of commas in writing large numbers. This is a traditional practice for writing numbers by hand, but in printed materials and calculator displays they are omitted. Students need to be encouraged to insert commas when they write down large numbers, as it helps them 'keep track' of the thousands and millions and thus read the number correctly.

Question 5 Answers will vary but for a pulse rate of 70 beats per minute the result is 36 792 000 beats per year.
Skills and knowledge: Number of minutes in an hour, hours in a day, days in a year; selecting the appropriate operation. If your participants are confident you may wish to extend the problem by asking the group how many beats there would be in a lifetime of 75 years and assist those who need to understand how the calculator deals with very large numbers.

Question 6 Answers will vary according to individual preferences and needs. From a purely mathematical viewpoint, the savings on the 2 litre bottle would be $0.34 for each litre of oil used.

Skills and knowledge: metric units; finding an appropriate quantity for comparison; selecting the appropriate operation; rounding decimals; interpreting an answer according to one's needs and preferences.

Question 7 Answer: about eleven and a half days (278 hours rounded to the nearest hour).
Skills and knowledge: number of seconds in a minute, minutes in an hour etc; working with large numbers; deciding on the appropriate operation; rounding decimals; making sense of the answer. Most people have difficulty getting a feel for large numbers. The calculator provides us with a ready tool for working with large numbers and getting used to them.

Question 8 Answer: $189.96
Skills and knowledge: percentages, using the % key (you may need to explain its use, as most people are not familiar with it); rounding decimals.

Question 9 Answer: $US1528
Skills and knowledge: For the first part of the problem, our expectation that we should get less US dollars than Australian dollars should guide our approach to selecting the appropriate operation for solving the problem, and help us check whether our answer is reasonable. For participants who ask for a way of explaining this problem clearly to numeracy students, here is an approach:

First, establish that 76.4 cents = $0.764

$A1 = $US 0.764

thus $A100 = 100 x $US 0.764 = $US 76.40 (i.e. 100 'lots of ' $US 0.764)

and so $A2000 = 2000 x $US 0.764 = $US 1528

Question 10 Answer: $220.30
Skills and knowledge: Reading and interpreting a table of information; selecting the appropriate combination of operations; checking calculations. This problem also provides you with the opportunity to discuss how the calculator's memory can be used.

Conclusion

To round up the discussion, you may like to ask the following questions :

- *How should calculators be used in an adult numeracy class?*
- *What are some of the problems associated with using calculators and how can these be addressed?*

Ask for comments on the activity; perhaps by asking each participant to briefly state one thing they learned from this session about using calculators effectively. Finally, distribute the remaining copies of Handout 2.7A.

Using Calculators

For each question, make an estimate **before** calculating your answer.

Then discuss and make brief notes on :

- Whether or not the calculator was useful, and if so, how.
- The skills and knowledge needed to solve the problem **using a calculator** (e.g. basic number facts, deciding on the appropriate operation, making sense of the answer, rounding decimals).
- How you checked your answer.

Questions

1. Add $1.37 and 13 cents.

2. What is the cost of 2.5 metres of fabric at $7.95 per metre?

3. How many buses are needed to transport 241 workers to a union meeting? (Each bus holds 36 passengers.)

4. A shower uses an average of 16 litres of water per minute. If you cut your daily shower from 10 minutes to 3 minutes, how much water would you save in a year? How many buckets would this fill? (A household bucket holds about 10 litres.)

5. Select a member of your pair and measure their pulse rate. How many times would their heart beat in one year at this rate?

6. You are at the supermarket (with your calculator), trying to decide whether or not to buy a 2 litre bottle of cooking oil instead of the usual 750 ml bottle. The 750 ml bottle is $2.39 and the 2 litre bottle is $5.69. Is it worthwhile buying the larger container? Explain what **you** would do and why.

7. How long would it take for one million seconds to pass?

8. A 24% discount applies to the purchase of a fabulous new numeracy teaching package which normally costs $249.95. What is the discount price?

9. You have just won an overseas trip in a raffle, including $2000 spending money. You need to purchase travellers cheques in US dollars. How many US dollars would you be able to obtain with $A2000 ? (The current exchange rate is $A1 =76.4 US cents.)

10. You are ordering supplies for the office, according to the following price structure :

Item	**Quantity**	**Cost**
Computer disks	Box of 10	$29.95
Photocopy paper	Ream (500 sheets)	$7.95
Highlighter pens	1	$1.45
Yellow sticky labels	1 packet	$1.95

What is the **total** cost of the following order: Four boxes of computer disks, eight reams of photocopy paper, a dozen highlighter pens, and ten packets of yellow sticky labels?

Problem Solving

What is Problem Solving?

Problem solving is the application of mathematical skills and reasoning to problems encountered in everyday life. Such problems range from the simple to the complex, for example:

- working out how much medicine to give a child and measuring it correctly;
- using or interpreting statistics at work to predict demand for goods so that an order can be placed with a wholesaler.

Such real-life problems never appear as a neatly ordered and graded page of straightforward 'sums'. To solve real-life problems we need to be able to draw upon our mathematical skills and use them to reach a solution which is reasonable and appropriate for the particular situation.

However, many basic maths students lack confidence with anything more complex than a worksheet full of sums which reinforce a single skill or concept, and are afraid to take risks and experiment with different approaches to solving an unfamiliar problem.

Many students have difficulty in deciding for themselves which operation or procedure is needed to solve a problem. For example a problem such as *How many 43c stamps can I buy with $5.00?* is difficult if students are unable to decide for themselves **how** to go about it.

Fear of making a mistake, low self-confidence and little knowledge of the problem solving **process** all contribute to students' difficulties in applying their mathematical skills and knowledge to non-standard or real-life problems.

The dangers of teaching mathematical **procedures** at the expense of mathematical **reasoning** are now well documented. For example, a national U.S. survey (Burns, 1990) found that about 80% of 17 year olds could calculate the answer to 3.04 x 5.3 = ? (answer 16.112). However, less that half that number could select the correct alternative to the same question when asked to select the most appropriate **approximation** from the following alternatives:

a) 1.6
b) 16
c) 160
d) 1600
e) I don't know

This question examined students' abilities to **reason** that if you multiplied a bit more than 3 by a bit more that 5 the answer would be about 16. That is the only answer from the choices that makes sense, yet only one in three students successfully solved the problem.

Problem solving activities are one way of bridging the gap between the numeracy class and the real world, so that students develop skills and strategies for solving a wide range of different types of problems.

What is a Problem Solving activity?

In the context of adult numeracy, a problem is a question or activity which is challenging to the student. Some students may be challenged by calculating change from shopping, whereas others would need a much more complex problem if they are to be challenged in any significant way. A question becomes a problem when the **answer is not immediately obvious** to the student; it requires more than a guess or a simple calculation to reach a solution. There are a number of other criteria which help define a problem solving activity.

- The **method of solution is not immediately obvious** and students need to decide for themselves how to solve it. This may be a simple decision about whether to divide or multiply or a systematic application of a number of problem solving strategies.
- There are a **number of different approaches**, or strategies, which lead to a solution.
- There may be **more than one correct or reasonable solution**. For example, a question such as *how many ways can you make $1 using 5c, 10c, 20c and 50c coins?'* Or asking students to brainstorm all the words that describe a shape. (This encourages creativity and divergent thinking).
- **There is more than one step to a solution.** For example, *'my age this year is a multiple of 7. Next year it will be a multiple of 5. I am not yet 50. How old am I?'* In order to solve the problem the student has to work out the multiples of 7, then the multiples of 5 and finally match them to reach the solution of 14 or 49.

How do students benefit from Problem Solving?

Problem solving activities in a numeracy class benefit students in two ways. First, students learn **about** problem solving. Problem solving activities introduce them to the problem solving process itself; the range of strategies or approaches that can be used to solve an unfamiliar problem. For example, to find all the numbers you can make with the digits 1, 2 and 3 requires students to be thorough and systematic in their approach. This is an important problem solving skill.

Second, problem solving activities can be used to introduce new concepts or reinforce existing ones. Take for example, the following problem: *If A=1, B=2, C=3 etc. how many points is your name worth?* This problem involves matching letters with numbers and gives students practice in addition. To solve a challenging extension of this problem: *can you find a word worth 50 or 100 points*? students need to try out different words until they are successful. Thus students are learning the age-old strategy of 'guess and check' and of course, persistence. Meanwhile students also get lots of practice in addition!

New concepts can be introduced to students using problem solving activities. The spatial problems in Handout 2.8B which follow use this approach for introducing ideas such as symmetry and volume. For further discussion of the opportunities problem solving activities provide for exploring underlying mathematical concepts see Jeannette Thiering's (1989) article in *Good Practice in Adult Literacy*.

How to begin with Problem Solving

The best foundation for success with problem solving is a positive and supportive learning environment where students feel comfortable about taking risks.

Teachers should acknowledge students' existing problem solving skills, perhaps by reminding them that everyday life is a constant process of problem solving. Take for instance the following everyday situations:

- *Have I got enough money to catch the train and buy lunch?*
- *How much meat should I buy for dinner tonight with two extra guests?*
- *What time should I leave home to get to school in time if I have to pick up a video on the way and drop my child off at creche?*

All these involve mathematical skills such as addition, subtraction and estimation, and problem solving strategies such as planning and carrying out the plan in a systematic way. Activities done at home such as fixing the lawnmower or making a cake need to be done in steps or stages (and in the right order!). A step-by-step approach is also a vital skill for solving more complex mathematical problems.

To encourage students with problem solving the teacher needs to model the problem solving process, by exploring and discussing different approaches, including ones that may not lead to a solution. This is important because many students, afraid of making a mistake, and believing that maths problems have instant solutions, give up very quickly. If, however, they see the teacher trying different approaches and finally reaching a solution they will be more likely to 'have a go' themselves. The matchstick puzzles in Handout 2.8B provide plenty of opportunities for encouraging this kind of experimentation. Using real matches also encourages students to take the risks they may be reluctant to take if their only tools were pen and paper.

Many students are still firm believers in the myth that there is only one 'proper' way to solve a problem. For example, even an apparently simple and straight forward problem such as *Can you make 30 cents using exactly 3 coins?* can be done using the coins themselves, diagrams of coins or written numbers. Each approach is valid. This problem also has more than one correct solution which helps break down the myth that maths problems only ever have one answer.

Finally, and most importantly, problem solving is best done in pairs or small groups. Working together on problems encourages students to verbalise what they are thinking and doing; to discuss, explain and perhaps defend their method of solution. This verbal interaction of course enhances learning.

The Fundamentals of Problem Solving

Many books have been written about problem solving, but none with the adult numeracy student in mind. What we have tried to do here is produce guidelines which are appropriate to the needs of students who are challenged by fairly simple problems.

The process of problem solving can be divided into three phases:

- Understanding the problem.
- Solving the problem.
- Checking the answer.

Understanding the problem

Students need to start by reading (or listening to) the problem very carefully to make sure they understand what they are being asked to do. This may involve:

- several readings (or repetitions by a speaker);
- thinking about the problem for themselves for a few minutes;
- discussing it with others in a small group and **reaching agreement** on what it is before they start;
- writing the problem down using their own words, symbols or diagrams;
- restating the problem verbally using their own words.

Once students have a clear idea of **what they know** and **what they need to find out** they are ready to begin solving the problem.

Solving the problem

Solving a problem involves exploring different approaches and finding one which successfully solves the problem. Strategies to focus on with numeracy students are listed below. These are summarised in Handout 2.8A *Problem Solving Guide*. This handout can be used to discuss problem solving strategies with students.

1. Use hands-on materials

The value of manipulative aids encourages risk taking and therefore cannot be over emphasised. Pieces of paper or objects such as counters or matchsticks which can be moved around encourage students to try out different combinations. The numbers in circles problems which follow can be solved in this way, using numbered counters or pieces of card.

2. Draw a diagram

A diagram or sketch can help students visualise and organise the information in the problem, and also provide a record of their solution process. For example, to solve the money problems which follow students may find it helpful to draw the coins themselves and label them.

3. Guess and check

This strategy involves making a guess and seeing where it leads. For example, to solve the problem: 'Can you make seven cents using exactly three coins?' students need to start by exploring different combinations of coins until they find one that works.

4. Organise the information

When exploring a problem, students need to work systematically. For example, to solve the problem: 'how many squares?' (see the problem solving activities which follow) it is a good idea to start with the smallest squares first and then proceed systematically through the different sizes. Lists, charts and tables are a powerful way of organizing seemingly confusing information. They encourage students to be systematic and to keep track of important information. For example, the problem 'what day will it be the day after tomorrow if four days ago was Friday?' is best solved if students start by making a list of the days of the week.

5. Look for patterns

Awareness of patterns and relationships is fundamental to learning and understanding mathematics, as well as a valuable problem solving strategy. Take, for instance, the nine times table:

1 x 9 = 9	7 x 9 = 63
2 x 9 = 18	8 x 9 = 72
3 x 9 = 27	9 x 9 = 81
4 x 9 = 36	10 x 9 = 90
5 x 9 = 45	11 x 9 = 99
6 x 9 = 54	12 x 9 = 108

The patterns and relationships include:

- the sum of the digits in the answer is 9;
- there is a descending and ascending pattern of numbers in the answers;
- the initial digit in the answer is one less than the number of 9's (up to 10 times 9).

Awareness of these patterns simplifies the task of learning and remembering. In order to solve the number patterns and magic squares problems which follow, students need to look for such patterns and relationships.

6. Break the problem into parts

To solve many problems, more than one step is required. This involves breaking the problem into parts and solving each part in turn.

For example, solving the magic squares problems (see following problem solving activities) requires deciding first what the numbers add up to and then filling in the rest of the squares according to the pattern.

7. **Make it simpler**

For adult numeracy students, making it simpler usually means replacing the 'hard' numbers in the problem with easier ones to make sure they are on the right track. For example, to solve a problem such as:

> *A monthly ticket is $53 and a daily ticket is $3.10. Which is cheaper if you use the train five days a week?*

This problem involves deciding which operation to use to work out a solution. Rounding off the numbers first can help students get a rough answer which will reassure them that they have used an appropriate method. For example, rounding the $53 to $50 and either dividing or multiplying by 20 (the approximate number of days in a month of travelling) enables students to see that dividing by 20 ($2.50) gives a more likely answer than multiplying ($1000). Once students are confident with their **method** they can go back and complete the problem using the exact figures and probably a calculator. Working the other way, rounding the $3.10 to $3 and multiplying by 20 (rather than dividing) gives an approximate answer of $60. Estimation is a fundamental numeracy skill and an invaluable aid to problem solving. Estimation prior to solving the problem gives an indication of the size of the answer and helps the student decide whether their final answer is reasonable. This is essential when using a calculator where it is inevitable that wrong keys will be pressed by mistake.

Checking the Answer

Checking the answer is an integral part of problem solving; a problem is not really solved unless the solution has been checked or verified. Returning to the examples of the lawnmower and the cake, the final test is whether the lawnmower starts and the cake is moist or rises properly. Students who can check their own work are also becoming independent and autonomous learners.

Students should start by re-reading the problem to make sure they answered the actual question asked. For example, a solution to 'How many ways can you find to make one dollar using other coins? ' should include a statement about how many different ways were actually found, and not simply a record of one or two possibilities. For open-ended questions like this one, students also need to think about whether they have included all possibilities, and how to tell whether or not they have found them all.

Calculations also need to be checked. 'Does my answer make sense?' is a question all students should ask themselves, and have the skills to answer. Students should be encouraged to solve the problems using another method in order to check their answer.

The following problem illustrates the importance of producing a sensible answer:

> *You are responsible for ordering mini-buses to transport a group of workers to a big union meeting. A mini-bus seats 14 and you have 119 workers to transport. How many buses will you need to order?*

Producing a reasonable answer to this question needs an appreciation of the situation. If a student uses a calculator to divide 119 by 14 the answer they will get is 8.5, which would not make sense. Students with an appreciation of the situation would realise that they need to order 9 buses.

The information in this section is presented in summary form in Handout 2.8A *Problem Solving Guide*.

References

Burns, M. (1990) The Maths Solution: Using Groups of Four. In Davidson, N. (ed. 1990) *Cooperative Learning in Mathematics: A Handbook for Teachers.* Menlo Park, California. Addison Wesley.

Thiering, J. (1989) Problem Solving and Mathematical Literacy, *Good Practice in Adult Literacy* , Vol 1, No 5, September 1990, p. 10-12.

The Activities

There are many problem solving activities in this kit which provide opportunities for discussing problem solving.

Handout 2.8B *Problem Solving for Numeracy Students* contains a selection of problems suitable for basic numeracy students, with accompanying discussion notes, which can be given out for participants to take away and use with their students.

There are a number of other problem solving activities in the kit.

Activity 4.4 ***Problem Solving Stations*** explores the use of hands-on materials for problem solving.

Activity 4.6 ***Cooperative Logic*** develops problem solving skills using structured group work.

Activity 4.7 ***Exploring Group Work with Problem Solving*** examines group dynamics and includes a range of problems which can be solved in group situations.

Problem Solving Guide

1. Understand the Problem

Make sure you understand the problem and what you are being asked to do. The following strategies may help.

- Read (or listen to) the problems carefully, several times if necessary.
- Think about it for yourself for a few minutes.
- Discuss the problem with others and reach agreement on what to do.
- Write the problem down (or say it aloud) using your own words.

2. Solve the Problem

Explore ways of solving the problem until you find an approach which works. Strategies to use include:

- Use hands-on materials
- Draw a diagram
- Guess and check
- Organise the information (eg lists, charts or tables)
- Look for patterns
- Break the problem into parts
- Make it simpler (e.g. by estimating)

3. Check the Answer

A problem is not fully solved until the solution is checked.

- Re-read the problem.
- Make sure you have answered the question asked.
- Check calculations.
- Check solutions using another method.
- Check that your answer makes sense.

Teacher Notes to Accompany Problem Solving for Numeracy Students

This collection of problems may be used in a variety of ways:

- as **stations** - see Activity 4.4 *Problem Solving Stations;*
- as **group problem solving activities** - see Activity 4.7 *Exploring Group Work with Problem Solving* ;
- as **individual activities** for students in the classroom;
- as **problems of the day/week** on the wall for students to attempt when they have spare time;
- as a **common activity** to begin the class;
- as a **break from other topics** during a lesson.

They are laid out so that you can cut them up and use them one or two at a time because presenting whole sheets of problems to students can be overwhelming.

Within this collection is a selection of problems which do not require particular numerical skills. This means that members of your class with differing skill levels will then be able to work together. Other problems are useful for students to practise their number skills, and we have also included a large number of problems using spatial skills because they provide one way to develop this often neglected area of numeracy education: the concepts and language involved in visually interpreting the three dimensional world in which we live. Skills such as interpreting two dimensional plans, following instructions for sewing or building, using maps and giving directions, interpreting words such as symmetry, square, triangle, are all improved by hands on experience through simple problem solving activities.

The problems have been arranged in categories to make it easy for you to select a diverse range or alternatively to concentrate on a particular skill area. Notes on the problems containing suggestions and discussion on each category are provided below.

The categories are:

Number Problems

Money Problems

Spatial Problems 1 (involving 2 dimensions)

Spatial Problems 2 (involving 2 and 3 dimensions)

Time Problems

Number Problems

- The **Number Patterns** selected here are simple and straightforward and designed to encourage counting in 2's, 3's, 10's etc. both forwards and backwards. These problems can serve as models for you to make up others of similar type if your students need more practice.

- The **Numbers in Circles** problems encourage students to practise their addition facts. If you use counters or beans it is easy to make up many others to supplement the examples given. Getting students to make up puzzles like this of their own can also be good practice at the number skills and encourage even more problem solving skills to be developed.

  Other similar problems can be found in chapter 4, section 2 of the kit, *Using Hands On Materials* - Activity 4.4 *Problem Solving Stations* (p 188).

- The **Coded Alphabet** problems serve many purposes: introduction to the idea of code; practice of addition; vocabulary and word practice; and interaction with other class members.
  You could extend the names exercise to find which class member has the highest/lowest number of points in their name. Finding the 100 point word could be a group exercise or one which challenges the class over a period of time, to fill in occasional spare moments, or to encourage students to try out words at home.

- **How Old am I?** is a typical problem solving activity. You need to ensure that students understand the concept of multiple. You could make up similar problems to reinforce the concept and the method.

- **Handshakes** is a simple version of another typical problem solving activity. Solutions can be found by acting it out or drawing diagrams. It can be extended to 4 people, 5 people, all the people in the room, or larger numbers of people. Advanced students could look for patterns occurring in the solutions which would allow them to work out the number of handshakes for any number of people.

- **Arranging the Digits** can be used to encourage a systematic approach to problem solving or just to practise distinguishing numbers from each other e.g. 231 is different from 213.
  A hint: if you have all the numbers they should add up to 1332. Try it on a calculator.

Spatial Problems

The rationale for concentrating on spatial skills is presented in the background notes in section 5 of this chapter: *Space and Shape* (p.59). The problems presented here are a collection of 2 and 3 dimensional problems, some of which introduce simple concepts, such as symmetry, as part of the problem.

All of these problems should be presented with accompanying hands on material, for example real matches or wooden toothpicks, wooden or plastic cubes or cut up shapes, which can be manipulated by the students as they solve the problem.

Money Problems

The problems selected here provide an opportunity for students to practise coin recognition, addition of money, and for more advanced students, systematic methods of problem solving.

When students are attempting these problems a supply of real coins should be available especially for those needing work on identification of coins.

The problems of the type 'how many ways can you do this ?' are open ended and can be tackled at many levels: advanced students might want to look for an exhaustive list but others may find a few ways and thus experience success at their own level.

If students enjoy these problems you can generate more of this type easily yourself or alternatively ask students to write some of their own for the rest of the group to solve. The latter suggestion is effective for integrating aspects of literacy and numeracy.

Time problems

Time poses different types of arithmetical problems for students. Having conquered addition in the decimal or base 10 system they have to turn their minds to slightly different frameworks - 7 days in a week, 60 minutes in an hour, and 24 hours in a day. These require a different way of thinking. It is therefore worth spending time on such calculations which occur so often and are so important in work and domestic life.

In this section we present only a few problems but they can serve as a model for developing others of your own. When embarking on time problems with students it is important to have concrete aids such as a variety of calendars and clocks for them to use. When students are clear about using the real thing, gradually encourage them to visualise, or draw diagrams so that they can progress to questions of similar type, using pen and paper jottings instead of needing the actual objects beside them.

Problems Solving for Numeracy Students

Number Problems

Number Patterns 1

What numbers are missing ?

2, 4, 6, 8, 10, _

3, 6, 9, 12, _ , 18, _

12, 11, 10, 9, 8, _ , _

20, 18, 16, 14, _ , 10, _

Number Patterns 2

Fill in the missing numbers.

10, 20, 30, 40, _ , _

51, 61, 71, 81, _ , 111, _

5, _ , 15, _ , 25, 30, _

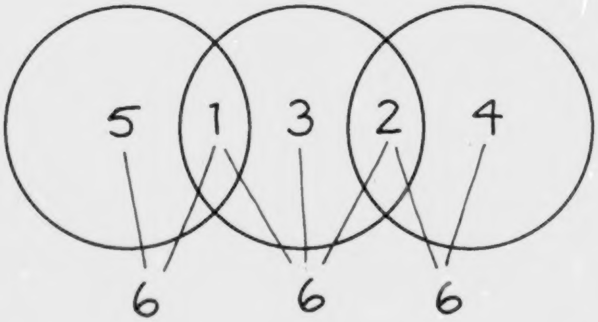
_ , 106, 206, 306, _ , _

100, 90, 80, _ , 60, _ , _

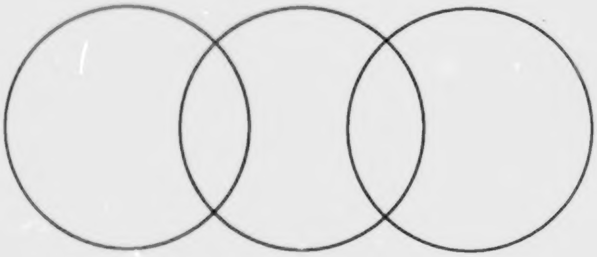
Number Problems

Numbers in Circles

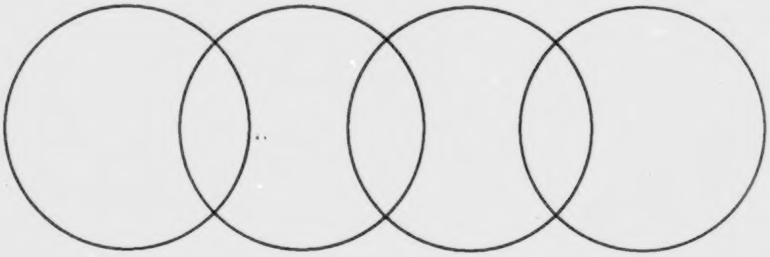
All the numbers inside each of these circles add up to 6



Try these:
Put the numbers **2**, **3**, **5**, **7**, **8** into the circles
so that inside each circle the numbers add up to $\boxed{10}$

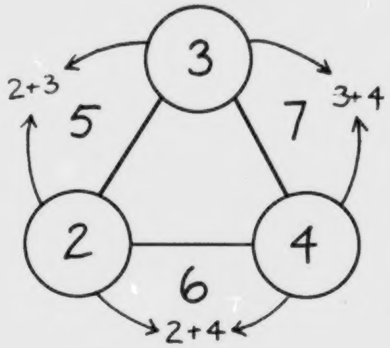


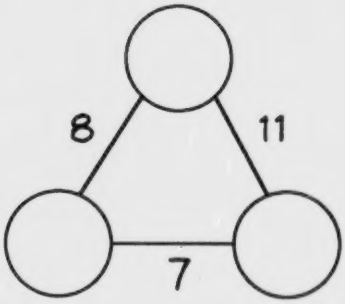
Use **3**, **4**, **5**, **6**, **7**, **8**, **9**, in these to total $\boxed{15}$ in each circle.

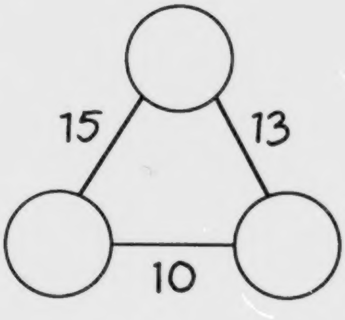


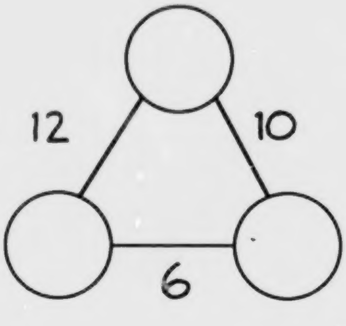
Number Triangles

Put numbers in the circles so that each pair adds up to the number between them.
For example:


3
2+3
5
7
3+4
2
4
6
2+4







Make up one of these puzzles for a friend to solve.

Number Problems

Magic Squares

In these squares all the **rows, columns** and **diagonals** add up to the **same** total.

8		
	5	7
		2

Use all the digits from 1 to 9
In other words,
1, 2, 3, 4, 5, 6, 7, 8, 9.

2	4	6
		1

Use all the digits from 0 to 8.
These are :
0, 1, 2 , 3, 4, 5, 6, 7, 8

	2	
	6	8
	10	

Use all the digits from 2 to 10

		11
8	13	6

Use all the digits from 5 to 13

Number Problems

The Coded Alphabet

If A = 1, B = 2, C = 3, D = 4, etc.

How many points is your name worth?

Is your name worth more points than
the person you are sitting with?

Can you find a word worth 50 points?

Can you find a word worth 100 points ?

GROUP PROBLEM

How Old am I ?

My age this year is a multiple of 7.
Next year it will be a multiple of 5.
How old am I?
(multiples of 7 are 7, 14, 28, ... etc)

(multiples of 5 are 5, 10, 15, ... etc)

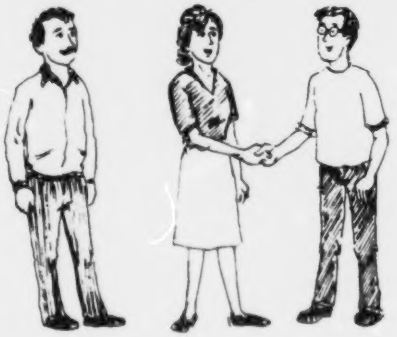
Number Problems

GROUP PROBLEM

Handshakes

If Maria, Joe and Tony meet
they all shake hands with each other.

How many handshakes are there ?



GROUP PROBLEM

Arranging the Digits

Find all the numbers you can make
with these three digits (single numbers):

1 2 3

Extension: How many numbers can you make
with the 4 digits:

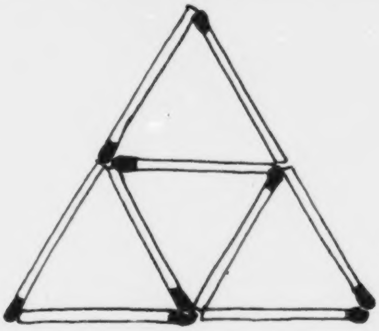
1 2 3 4

Spatial Problems 1

Stick Puzzles 1

(using toothpicks, matches or icy-pole sticks)

With 9 sticks make this shape:

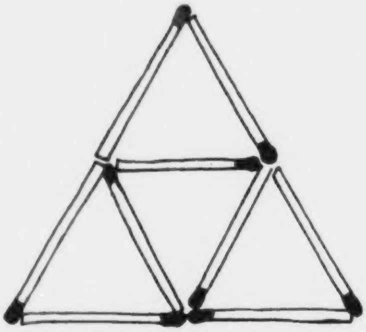


Can you take out 2 sticks and leave 3 triangles?

Try and remove 6 sticks and get one triangle.

Stick Puzzles 2

Make this shape with 9 sticks



Can you take out 3 sticks and leave 1 triangle?

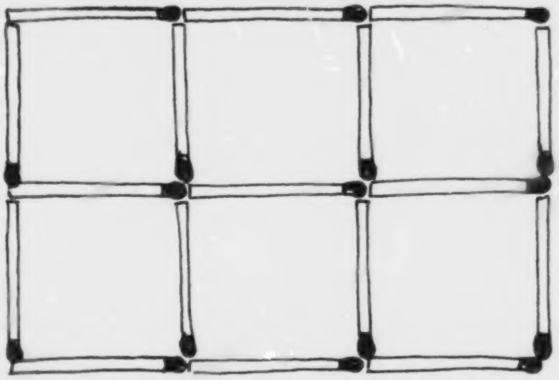
Now remove 4 sticks and leave 2 triangles.

Try taking 2 sticks and leave 2 triangles.

Spatial Problems 1

Stick Puzzles 3

Make this shape using 17 sticks.

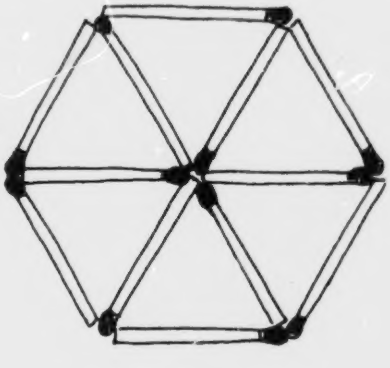


Can you take out 5 sticks and leave 3 squares?

Can you take out 6 sticks and leave 2 squares?

Stick Puzzles 4

With 12 sticks make this shape.

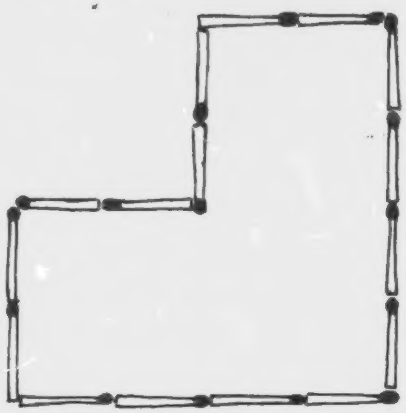


Now try to take out 4 sticks and leave 3 triangles.

Spatial Problems 1

Stick Puzzles 5

A farmer has land this shape. Make it with 16 sticks.

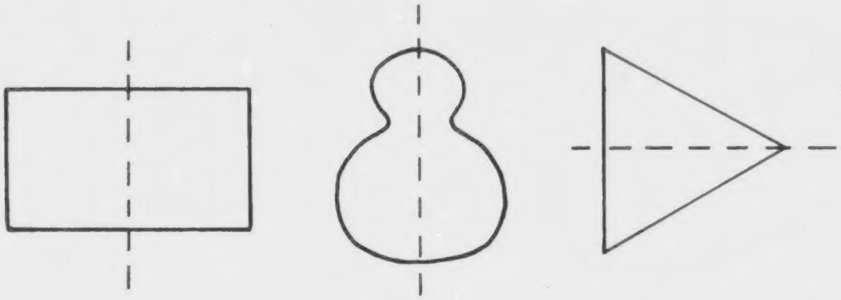


She wants to divide it exactly in 3 pieces
for each of her 3 children.
Can you help her by adding 4 sticks?

Spatial Problems 1

Symmetry

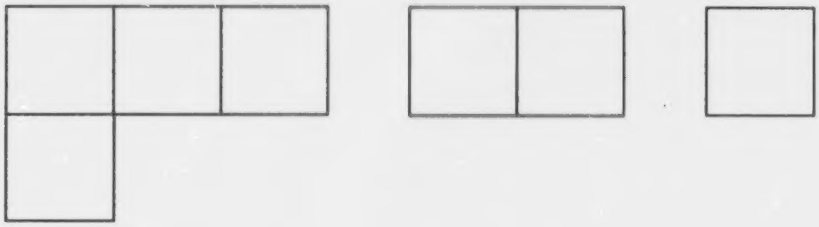
These shapes have **symmetry**



because they can be cut or folded into
two pieces exactly the same.

Can you put these three pieces together
to make a shape with symmetry?

Cut them out and try.

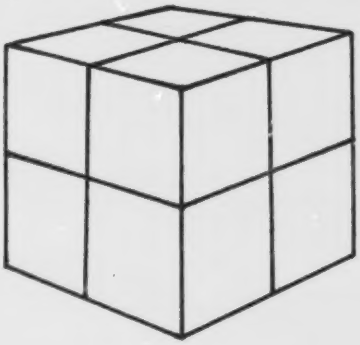


How many other symmetrical shapes
can you make with them?

Spatial Problems 1

Cubes

Using small blocks make a cube
that has 2 small blocks along each side.

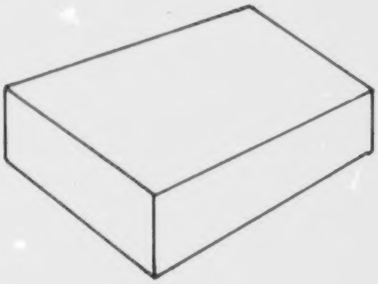


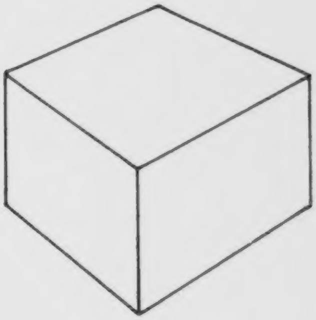
How many blocks did you use?

A **Cube** is the same size on all sides.
Can you make a **Cube** using:

27 blocks?	16 blocks?

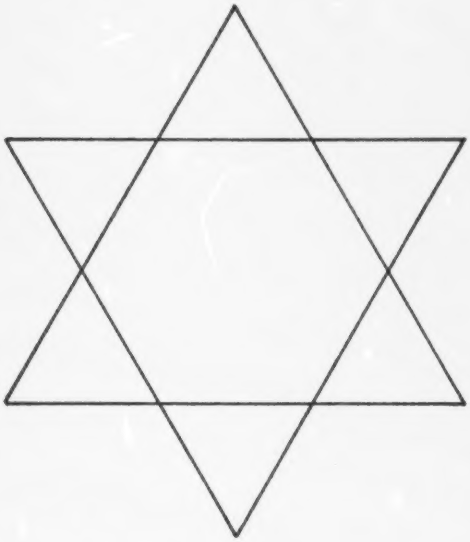
Can you make a box shape with 24 small blocks?



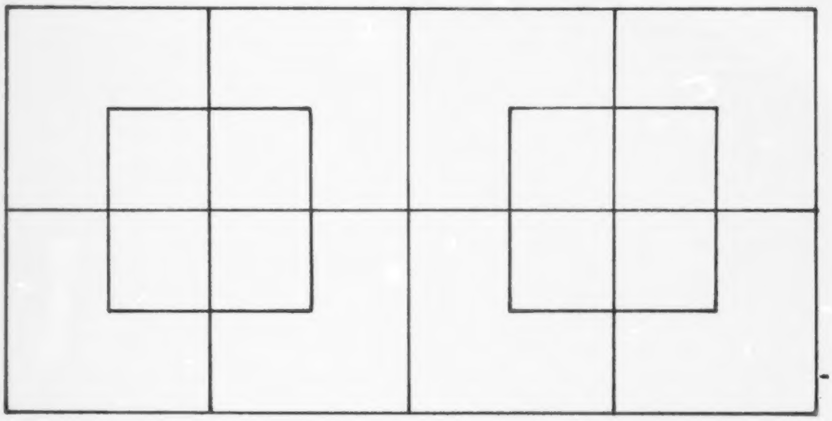


Spatial Problems 1

How many triangles can you see in this picture?



How many squares are in this figure?

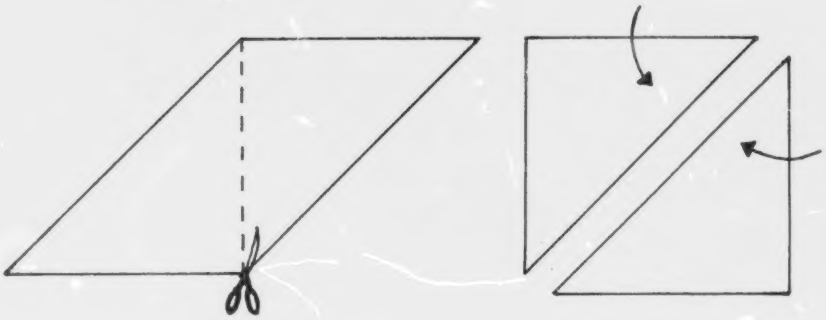


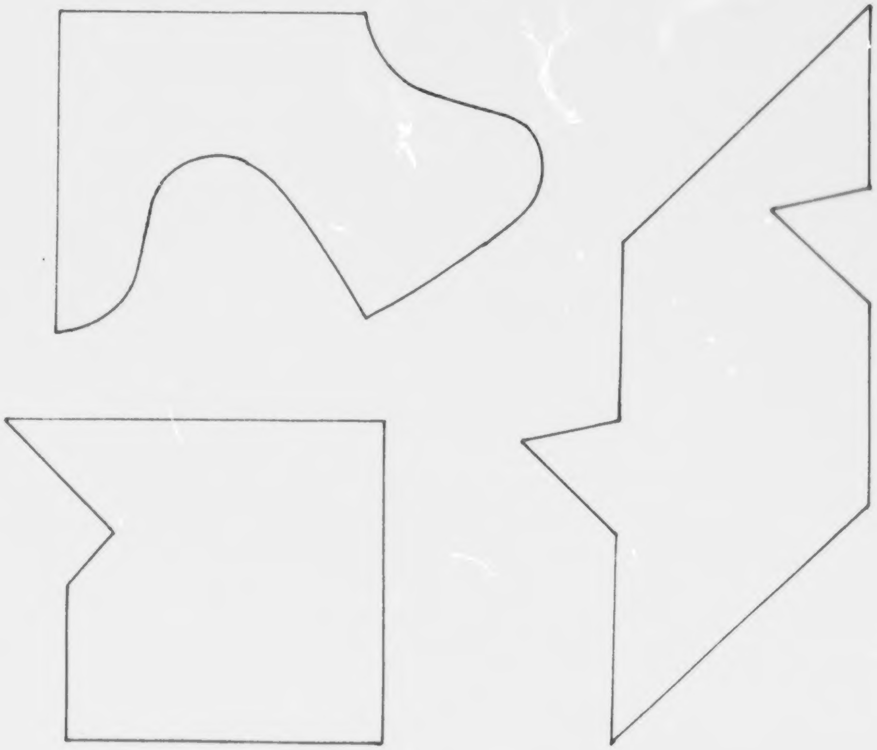
Spatial Problems 1

The Square

Cut out the 3 shapes below.
Then for each one try to make a straight cut so that the two pieces fit together to make a square

Like this

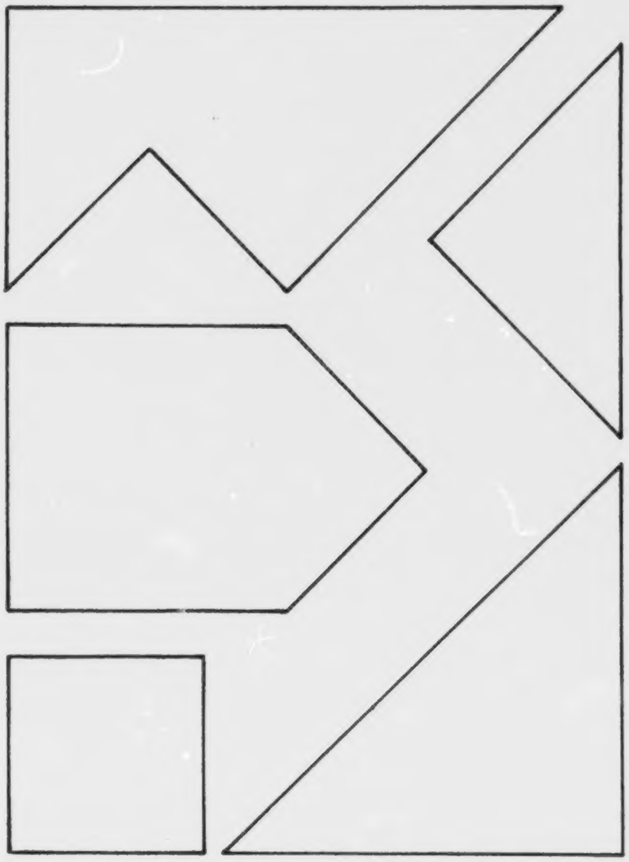




Spatial Problems 1

Making Shapes - Saving Waste

To save wasting material Rosza found a way to put these pattern pieces together in one square. Can you do it? (Cut them out and move them around.)



Can you make a different square using only four of the shapes?

Based on a problem by Hannah Schevach

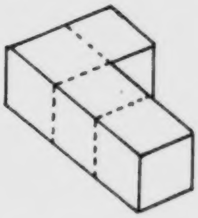
Spatial Problems 2

Rotating Shapes

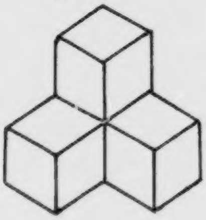
Many shapes look different if you turn them around.

Each shape here is made of 4 blocks

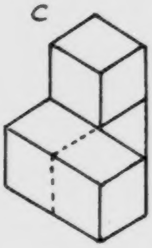
A



B

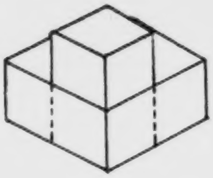


C

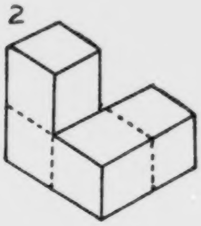


and has been turned around to make one of these:

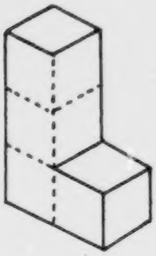
1



2



3



Can you match them ?

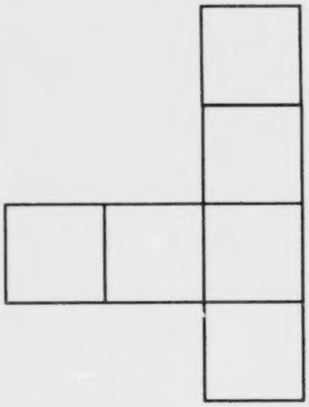
Make the shapes from the blocks to check.

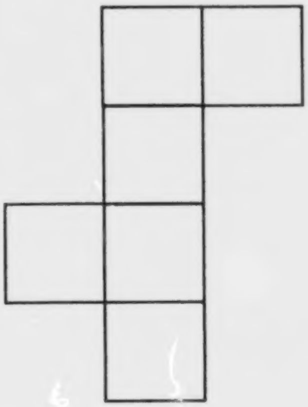
Handout 2.8B p.19

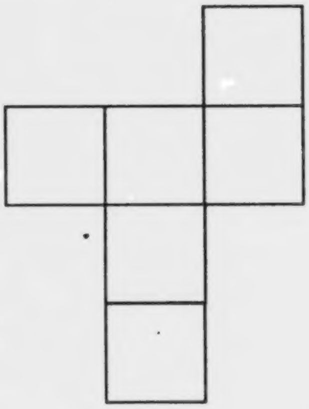
Spatial Problems 2

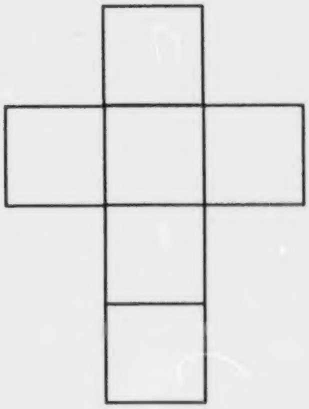
Which of these shapes can be folded
into a closed box (cube)?

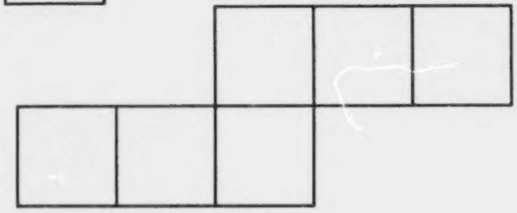
Guess first then cut them out and see if you are right.







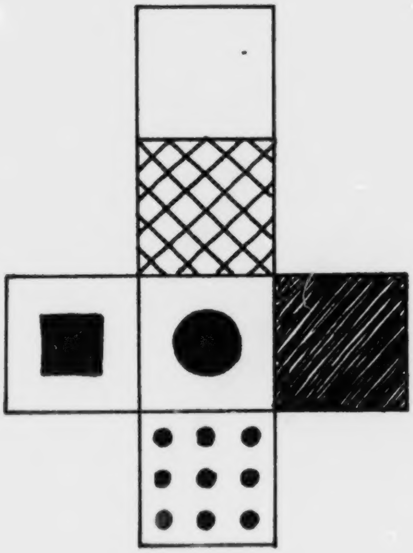




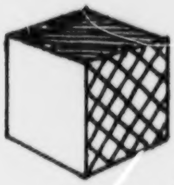
Spatial Problems 2

The Wrong Box

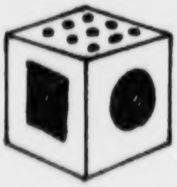
If this shape



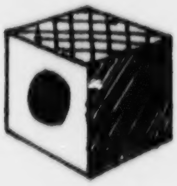
is folded to make a box, which one of these will look wrong ?



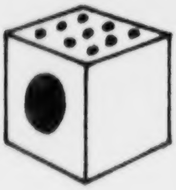
A



B



C



D

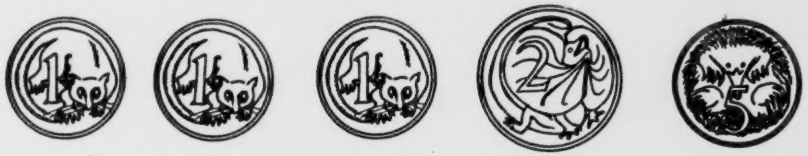
Handout 2.8B p.21

Money Problems

Coin Problem 1 - Ten Cent Problem

Use smaller coins to make exactly 10 cents

One way is



How many other ways can you do this?

Coin Problem 2 - Making Seven Cents

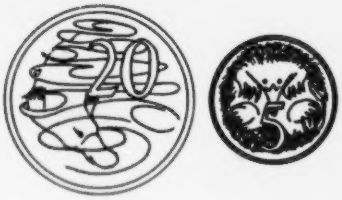
Can you make seven cents using exactly three coins?

then four coins?

and five coins?

Money Problems

Coin Problem 3 - Making 25 Cents



Make 25 cents

using exactly : 3 coins

4 coins

6 coins

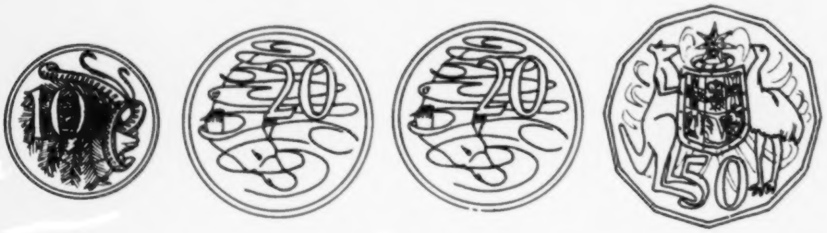
7 coins

GROUP PROBLEM

Coin Problem 4 - One Dollar

How many different ways can you find to make a dollar using other coins?

One way is



Keep a record on paper as you go.

Handout 2.8B p.23

Time Problems

Taking Your Medicine

A doctor prescribes nine pills.
You take one pill every half hour.
You take the first pill at 1 o'clock.
How long would they last ?

Mayday! Mayday!

What date will it be
in one fortnight
after Mayday
(May 1st)
this year ?

A Lucky Day?

If today was
Friday June 13th,
what would be
the date a week
from next Sunday ?

Time Problems

What Day Is It? - 1

What day will it be the day after tomorrow if four days ago was Friday ?

What Day Is It? - 2

What day will it be tomorrow if it was Saturday the day before yesterday ?

When Should Julie Go?

What time should Julie leave home?

- Her class starts at 9.00 am
- It takes 20 minutes to drive to college
- She has to drop her daughter off at creche (an extra 10 minutes)
- She has to pick up a video (an extra 5 minutes)
- She likes to arrive 10 minutes early for class.

Chapter 3. Building Positive Attitudes.

Many students involved in adult basic education experienced failure in the school system, and lost confidence in their ability to learn. This is particularly true for mathematics. Teachers of adults encounter many students with little self confidence, who are convinced that success with numbers and maths is beyond them. Some students experience real fear when confronted with numbers, a syndrome that Sheila Tobias examines in her book *Overcoming Math Anxiety*. She defined maths anxiety as the:

> *panic, helplessness and mental disorganization that arise among some people when they are required to solve a mathematical problem* (Tobias, 1978).

The U.S educator Dorothy Buerk unearthed some very strong feelings about mathematics which were expressed by one of her students in a powerful and imaginative way:

> *On the eighth day, God created mathematics. He took stainless steel, and he rolled it out thin, and he made it into a fence, forty cubits high, and infinite cubits long. And on this fence, in fair capitals, he did print rules and pointed reminders. 'Invert and multiply.' 'The square of the hypoteneuse is three decibels louder than one hand clapping.' 'Always do what's in parentheses first.' And when he was finished, he said 'On one side of the fence will reside those who are good at math and on the other will remain those who are bad at math, and woe unto them, for they shall weep and gnash their teeth'* (Buerk, 1985, p 59).

Even people with high achievements in other fields (particularly women) express feelings of inadequacy about mathematics.

The Australian playwright Dorothy Hewett, in her recently published autobiography, writes of her bad experiences of learning maths, which created a lifelong dislike and fear of the subject:

> *I can't add up or subtract. It's all a mystery to me but every year I win second prize for illustrated stories and poems at the Royal Show ...*
> *My father is clever at arithmetic. At Korong Vale school, when they couldn't teach him any more, he became a pupil teacher at thirteen. He tries to explain it all to me but his methods are extraordinary; masses of little black figures march mysteriously up and down the page. My mind goes blank.*
> *'Add it up,' he yells. 'Tell me the answer. Any baby could do it.' His dark face thrusts itself into mine, swollen with rage. 'I'll make you answer.'*
> *He throws me over his knee and starts beating me hard with the flat of his hand. I am outraged. Although it hurts I can't even cry* (Hewett, 1990, p. 39).

It is not difficult in this case to guess at the origins of her fear and dislike. Such stories of humiliation are not uncommon in adult mathematics classes.

What Causes Negative Attitudes to Maths?

Nearly every adult has a story about their experiences with mathematics, and unfortunately, most of the experiences were negative.

From listening to students' stories, it seems that negative feelings about maths have their origins in experiences in the classroom and at home and are reinforced by socialisation, particularly gender stereotyping.

Traditional Classroom Practices

Classes were often very large, and teachers unable to give students the attention they needed. Many students spent years in these classes understanding little of what was said by an apparently remote and impatient teacher who ignored them in favour of high achieving or misbehaving students. Some suffered humiliating experiences, such as being forced to the blackboard to explain to the class a problem which they were unable to do themselves.

Other traditionally accepted aspects of the maths class also contributed to fear of maths and loss of confidence. Maths was usually taught in a competitive manner, where collaboration was seen as cheating, rather than (as nowadays) a valuable opportunity for cooperative learning.

Emphasis on the right answer, rather than on the process of problem solving, and on memorisation, rather than understanding, made learning a difficult and alienating process for many. The 'sums' to which the teacher applied magical rules were impersonal and unrelated to people and human concerns, and had little to do with students' own lives and experiences.

Most adult students had little exposure to hands-on materials or visual aids. Teachers generally relied on abstract and verbal reasoning - or even moral arguments, such as 'borrow and pay back' - to explain mathematical processes. This disadvantaged students who would have benefited from visual and tactile experiences. In a traditional teacher-dominated classroom, students were also denied valuable opportunities to learn through verbalising their experience.

Timed tests, competitive mental arithmetic quizzes and the pressure to keep up with the rest of the class often created anxiety and blocked students' ability to think clearly.

Attitudes to Women

Women have been particularly disadvantaged. During school days there was little encouragement for most to persevere. Women who studied maths and science were considered 'unfeminine' and the absence of positive female role models further reinforced traditional choices. Even today, women in traditional classrooms feel excluded from mathematics. Australian educator Mary Barnes has documented the attitudes of women students and notes that they find that what is taught is irrelevant to their interests, too abstract and impersonal and unrelated to people or human concerns. They comment that it is competitive, logical, rational and unemotional and the way it is taught does not suit their learning style (Barnes, 1988).

A Barrier to Opportunities

Exclusion from maths is an important issue because a lack of mathematics skills limits people's choices. Mathematics-based selection tests are widely (and inappropriately) used by employers. Applicants who fail these tests are thus 'filtered out' from the maths literate, irrespective of their ability to handle the demands of the job, which does not necessarily include maths. This process also occurs in selection for many TAFE and tertiary courses, whose curriculum does not necessarily include mathematics (e.g. social sciences). Technical and scientific tertiary courses are of course inaccessible to students without mathematics prerequisites. Since about twice as many males as females in Australia attempt advanced mathematics courses in the final year of secondary school, it comes as no surprise that women are under represented in these courses.

Because of its role as a hurdle requirement, or 'critical filter', skill in mathematics is often falsely equated with 'intelligence' or 'ability'. People who fail maths thus tend to believe that they also lack intelligence, and as a result suffer a loss of confidence and lowered expectations of what they can achieve academically, whether mathematical or not.

Building Positive Attitudes in the Classroom

Many students experience considerable anxiety about returning to study maths, and it is important that these feelings are acknowledged by the teacher. It is often very helpful to conduct class discussions where students share their experiences of learning mathematics in the past and their present feelings about learning it. discovering that they are not alone comes as a great relief to students, creating new feelings of confidence and determination to succeed. Such discussions also help build group cohesion and support which is a vital aspect of the supportive learning environment in which students feel safe to take the risks necessary for learning.

The innovative teaching and learning activities discussed in chapter 4, such as cooperative learning, hands-on materials and games, also make a significant contribution to building positive attitudes to learning maths.

References

Barnes, M. (1988), Mathematics: A Barrier for Women? In Caine, B. de Lepervach, M. and Grosz, E. *Crossing Boundaries*. Sydney, Allen and Unwin.

Buerk, D. (1985), The Voices of Women Making Meaning in Mathematics. *Journal of Education* , Vol 167, (3) p 59-70.

Hewett, D.(1990), *Wild Card: An Autobiography 1923-1958*. Melbourne,McPhee Gribble.

Tobias, S. (1978), *Overcoming Math Anxiety*. Boston, Haughton-Mifflin.

The Activities

The activities in this chapter are designed to raise participants' awareness of their own learning experiences, and the factors which shape attitudes to mathematics. Confidence building activities are also presented for them to use with students.

Activity 3.1 ***Sharing Experiences of Maths*** is designed to set the scene of the program. It functions as an icebreaker, as well as a means for participants to discuss their experiences with mathematics and examine their emotional responses to mathematics, including their feelings about teaching numeracy. The activity then proceeds to a discussion about what makes a good numeracy teacher, resulting in a range of practical strategies for building students' confidence with numeracy. Included with this activity are two examples of student activities for building confidence with maths. Handout 3.1E *Stories with Cards* is an activity in which students talk about everyday numbers in an informal and creative way, using cards selected from a pack of playing cards. Handout 3.1F *Numbers in our Lives* provides practice with numerical language and also an opportunity for a group of new students to get to know each other.

These handouts can be distributed to teachers and tutors participating in any of the activities in this kit, as examples of student activities which provide a gentle introduction to work with numbers.

Activity 3.2 ***How Adults Learn*** starts by examining participants' experiences of learning as an adult, in order to pinpoint the factors which contribute to successful (and not so successful) learning experiences. Participants then focus more closely on the learning of numeracy, and develop and discuss a set of principles for teaching adult numeracy. We suggest that you select either of these two activities to do with your group. For a group whose confidence with mathematics is an issue (e.g. literacy teachers) we strongly recommend Activity 3.1.

For the mathematics trained (who generally experienced success with mathematics) Activity 3.2 may be more appropriate, as it includes discussion of a wider range of learning experiences, which are more likely to include areas with which mathematics teachers may have had problems. The important thing here is that teachers discuss something that was an area of difficulty for themselves, so as to better identify with the needs and difficulties of their students.

Activity 3.3 ***Experiencing Success with Metric Length*** performs two functions; whilst being a valuable lesson for teaching metric estimation it is also an example of an activity which can be done in class which ensures success for all students and thus enhances their confidence as learners.

Activity 3.4 ***Acknowledging Students' Skills and Experience*** demonstrates two examples of activities in which students can succeed because of the life experience they bring to the classroom. These teaching materials provide the starting point for participants to design a classroom activity themselves to use with their own students.

A note for presenters

Because of the widening of the definition of literacy to include numeracy, some literacy teachers are now being asked to incorporate numeracy into their programs. However, many literacy teachers lack confidence in their own maths skills, and in fact may have some maths anxiety of their own to overcome.

As mentioned previously many women (and most literacy teachers are women) who are successful in other fields feel inadequate and fearful about mathematics. That is one of the main reasons they participate in staff development, and it is important to acknowledge their feelings about the subject. Enough time needs to be allocated to activities so that participants do not feel rushed. Take care not to make assumptions about levels of knowledge and skill. For instance, if using calculators check first if there is anyone who is unfamiliar with them. If there is, ensure that you explain the basics or that they are paired with someone supportive who can explain the activity as it proceeds.

When conducting your workshop you are likely to have participants with very different levels of confidence, and this will influence the emphasis in discussions. Literacy teachers may tend to focus on their own maths anxiety, whereas numeracy teachers may tend to focus on the students' anxiety. In a mixed group the value of the discussion rests on what both sub-groups can learn from each other. Make sure that those who are less confident are not dominated by those who have a strong maths background.

Activity 3.1 **Sharing Experiences of Maths**

Rationale

Examining and sharing one's personal experience of mathematics is an excellent icebreaker, as well as a means of understanding how attitudes to mathematics develop. By comparing their experience with that of others, participants can identify what is unique and what is common in their experience and gain an understanding of the factors which determine their attitudes and feelings about maths. The activity also allows participants to discuss their feelings about numeracy teaching and pinpoint their strengths and weaknesses. Through group discussion, participants identify the qualities of an effective numeracy teacher, and develop strategies for overcoming anxiety and building positive attitudes in the basic maths classroom.

Aims

- To increase participants' awareness of their attitudes to and feelings about maths.
- To identify the origins of current attitudes to maths, both positive and negative.
- To identify and discuss participants' attitudes to teaching numeracy.
- To identify the qualities of an effective numeracy teacher.
- To develop strategies for building positive attitudes in students and a means for students to identify their learning goals.

Materials and Equipment

- Whiteboard or butchers' paper
- Pens

Time

45 minutes

Handouts

- 3.1A *My Experiences of Maths*
- 3.1B *Maths and Me* (Student Activity)
- 3.1C *Principles for Teaching Adult Numeracy*
- 3.1D *Strategies for Overcoming Maths Anxiety*
- 3.1E *Stories with Cards* (Student Activity)
- 3.1F *Numbers in our Lives* (Student Activity)

Preparation

Photocopy one copy each of Handouts 3.1A, 3.1B, 3.1C, 3.1D, 3.1E and 3.1F for each participant. Arrange furniture so that participants are seated at tables in groups of four to six.

Conducting the Activity

Briefly describe to participants the aims of the session, for instance by saying:

> *In this activity we will be discussing our own experiences in learning maths, and what shaped our current attitudes and feelings about the subject. We will then be in a better position to examine more closely our own role in teaching numeracy, and how to build positive attitudes in our own students .*

Part 1: Reflecting on our own experience

It is a good idea to warm up the group with some discussion of attitudes and feelings about mathematics, particularly negative ones, and the origins of these. Referring to the background notes, make a short presentation to the group, using the experiences of Dorothy Buerk and Dorothy Hewett and perhaps some material of your own. You may also like to ask the group for stories about their experiences of learning maths.

Distribute Handout 3.1A *My Experience of Maths* and ask participants to take about five minutes to reflect on their own experience and complete the sentences.

When participants have finished, ask them to share their responses with others in their group. One way for them to do this is to take each question in turn, and discuss the similarities and differences in their experiences and attitudes. Circulate among the groups and listen to the discussion, as this will help you pinpoint important issues. Allow the discussion to continue for about fifteen minutes.

Discussion

Bring the group together and discuss the main issues, perhaps by taking each question in turn. Some questions to start the discussion are listed below. You may like to refer to or quote from the introductory notes to add to the discussion.

Questions 1 and 2

- *What was maths like at school?*
- *How did your school experience shape your attitudes?* (e.g. content, teaching methods, teacher attitudes, peer attitudes)
- *What other messages did you get about maths, and where did these come from?* (e.g. parents, media)

Question 3

- *How do you feel about doing something that involves maths now?*
- *What is the relationship between your feelings now and your experiences at school?*

Question 4

- *How do you feel about teaching numeracy?* (It may be useful to record positive and negative responses on a chart, in two lists).

Refer to the list of 'negative feelings'

- *What do you think is needed to counteract these feelings?* (Record suggestions, which may include needs for further maths education, staff development, better resources and so on. These needs should inform and guide the numeracy staff development activities you plan to present.)

Question 5

- *What are some of the attributes of a good numeracy teacher?* (List responses.These should provide participants with a list of criteria for good teaching practices.)

Now distribute Handout 3.1C *Principles for Teaching Adult Numeracy*. Read it aloud, or give participants time to read it themselves. This statement is a concise set of guidelines for teaching numeracy to adults. Some questions for discussion include :

- *Are there any questions or comments about these principles ?*
- *How do these guidelines compare with your responses to Question 5 ?*
- *Are there further points you would like to add to this list ?*

A similar activity to the one just completed can be conducted with numeracy students. Distribute Handout 3.1B *Maths and Me* (Student Activity). This handout can be used with students as a discussion starter in much the same way as the activity just conducted with teachers. The activity also provides students with an opportunity to make suggestions on what they would like to learn. These suggestions can be discussed with the class, forming a starting point for a negotiated curriculum.

Part 2: Strategies for building students' confidence

This part of the activity focuses on strategies for enabling students to overcome their fears of maths and feel more confident in their ability to learn. The principles for teaching adult numeracy just discussed contribute significantly to this process. Specific strategies for overcoming maths anxiety can also be useful, particularly when students are feeling 'blocked', or difficulty in class triggers off old fears or memories.

Ask each group to spend five to ten minutes in discussion and to come up with two or three strategies which may help students overcome maths anxiety. (For experienced teachers, it would be most useful to discuss and share approaches which have worked for them.)

Bring the group together and discuss their ideas. Record these on butchers paper or whiteboard. Distribute Handout 3.1D *Strategies for Overcoming Maths Anxiety* and ask for comments, or more suggestions. Some further ideas include:

- a writing exercise in which students tell their 'maths story';
- students keeping a journal in which they write about how they feel about maths and what they are learning;
- students suggesting strategies for overcoming maths anxiety and making a list or poster as a group exercise. the poster could then be displayed in a prominent place in the classroom.

Structured classroom activities for overcoming maths anxiety can also be found in Marr, B. and Helme, S. (eds.1987) *Mathematics a New Beginning*. State Training Boaord of Victoria.

Two activities form Goddard, R. and Marr, B. (1991) *Strength in Numbers* have been included as examples of student activities for building confidence with maths. Handout 3.1E *Stories with Cards* is an activity where students talk about everyday numbers in an informal and creative way, using cards selected from a pack of playing cards. Handout 3.1F *Numbers in our Lives* provides practice with numerical language and also an opportunity for a new group of students to get to know each other. These handouts can be distributed to participants to conclude the session.

My Experiences of Maths

What I remember most about maths at school is

....

....

....

Most of my maths teachers were

....

....

....

When I have to do something that involves maths now I feel

....

....

....

When I think about teaching numeracy I feel

....

....

....

A good numeracy teacher

....

....

....

Maths and Me

Please complete these sentences:

Maths makes me feel

....

....

....

Maths at school was

....

....

....

I've never been able to

....

....

....

I'm good at

....

....

....

I'd like to learn how to

....

....

....

Principles for Teaching Adult Numeracy

- Ensure that all students in the group perform tasks at which they can experience **success.** This will build their confidence as they progress through their course.
- Encourage students to learn through **interaction and cooperation**. This involves discussion, asking questions, explaining their reasoning to others, and working cooperatively in pairs or small groups.
- Use **practical activities and hands-on materials.** Effective learning takes place when students start with their own experience and build on that experience to develop concepts and skills which they can apply in unfamiliar situations.
- **Negotiate the curriculum.** When determining course content and teaching methods be continually responsive to students' expressed needs and interests, and adapt the program accordingly.
- Teach concepts in a **context relevant to adult students**, drawing from their backgrounds, interests and experiences.
- Use **gender inclusive teaching strategies.** Include contexts which are familiar to women, as well as those which are neutral or familiar to men. Teaching resources should avoid sexist language, ensure equal representation of males and females in illustrations and examples, and show females and males in non-traditional roles.
- Encourage students to reflect on their learning, so that they become more aware of **how** they learn, understand and remember. This includes discussing with students the theories behind the methods of teaching used.
- Raise **awareness** about the social, economic and political systems influencing our lives. This can be done by developing examples which refer to data and information currently available in newspapers and magazines, on television and radio and in government publications.
- Acknowledge **differences between students** in their backgrounds and levels of mathematical skills. Materials should have the scope to be used at a variety of levels within a class and contain a range of reinforcing activities to be done at home or in subsequent sessions.
- Make learning an **enjoyable experience.** Many adult students come to basic education with memories of traditional classrooms and dull maths lessons. It is our job to reverse this by providing an exciting classroom atmosphere with a range of activities and teaching approaches which stimulate interest and discussion.

Handout 3.1D

Strategies for Overcoming Maths Anxiety

- Create a supportive group atmosphere in which students can relax and be themselves.
- Acknowledge and discuss students' feelings about maths.
- Use activities which provide an early experience of success.
- Encourage risk taking, rather than perfection in maths. (Accuracy is not the only criteria for success.)
- Value a range of approaches and methods of solving problems.
- Focus on what students are able to do, acknowledging their skills and experience.
- Encourage students to take their time and be persistent. (Time pressure can trigger anxiety.)
- Use enjoyable and motivational activities, including games.
- Use hands-on materials and activities.
- Include new and innovative teaching methods and materials. (Traditional approaches may trigger anxiety.)
- Use cooperative learning strategies, such as group and partner work.(Isolated learning can be stressful.)
- Teach in context, using situations and examples which are relevant to students' needs, goals and interests.
- Teach students how to use calculators and encourage their use.
- Expect the need to modify or abandon an activity on the spot if students become anxious.
- Help students identify their own strategies for overcoming maths anxiety.

Handout 3.1E

STORIES WITH CARDS

Skills developed

- Use of language related to numbers

Materials

- Pack of playing cards

This is a further session in which students talk about everyday numbers in an informal and creative way. It again gives an opportunity for students to get to know each other.

SESSION OUTLINE

Deal 4 cards to a student. Ask them to tell a story about those numbers.

Some of the examples we collected are:

8 2 A 3

One girl had her 8th birthday on the third of the second and it was ace.

8 10 J 9

One day Jack was walking down the street and went to the pet shop where he saw 8 guinea pigs, 9 rabbits and 10 fish.

A 5 10 6

Brazil was behind 5-6 in the volleyball with only 10 people watching. Brazil took the ball and served an ace.

Note : Although they can add interest, you may choose to remove the picture cards.

Extension

Ask students to include at least **one** or **two** metric units in their story.

eg. Jack rode 9 kilometres to the pet shop where be bought 8 kilograms of dogfood for his 10 puppies.

NUMBERS IN OUR LIVES

Skills developed
- Interaction within the group
- Practice with numerical language

This session is designed to enable students

- to get to know each other
- talk about everyday numbers in a non-threatening and interesting way - people are curious about each other.

This is also a good activity for you to observe students confidence with numbers.

SESSION OUTLINE

There are 2 activities to choose from or you may want to use both.

Activity 1 About Me
Go around the group and ask each member in turn to make a simple statement about themselves involving numbers. You should start the ball rolling with the first statement.

Some ideas are:
- *My uncle has only 8 toes*
- *I am 158cm tall*
- *My favourite number is 7*
- *I have 3 children*

There is fun to be had here when students warm to the idea and they may want to make more than one statement.

For 1-1 Tutors
You and your student each think of 10 facts about yourselves. Take it in turns to give a fact.

Activity 2 Questions About Others
Now you or the students can gather information about the rest of the group by asking questions.

Begin by making a few suggestions yourself, as some people will find it difficult to think of a question immediately. Discussion will stimulate ideas. The questions can be recorded on the board or butcher's paper, one question from each student.

Handout 3.1F p.2

eg.
- *How many times a year do you go to see a film?*
- *How many people live in your house?*
- *How many cups of tea or coffee do you have each day?*
- *How many years since you were at school?*
- *How many children do you have?*
- *What size family did you come from?*

Now that you have the questions you will want to collect the answers. This could be done in several ways:

1. You collect answers and record them on the board, discussing the results as they are collected.

2. Students could move around the room collecting answers to their own question, giving them experience in collating data.

   eg.

Name	Number of Films
Gary	10
Soula	6

   They would then come together and discuss their results.

3. Within the group the students ask their question and others answer verbally with discussion following each question.

Further Ideas

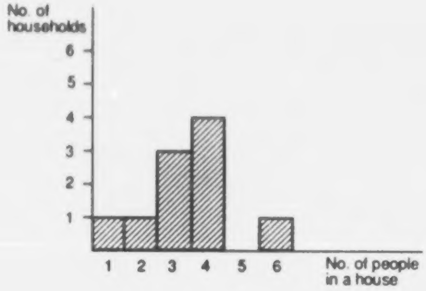
1. Questions such as the first one above can lead into a discussion about estimation.

2. Some questions can be used to introduce graphs and tables if it seems appropriate at this stage. Ask the students to imagine how their information could be represented pictorially.

   eg. In a class of 10 students, the responses to the question:

   How may people live in your house?

   could be recorded as:



Activity 3.2 How Adults Learn

Rationale

Examining and sharing one's experiences as an adult learner helps us identify how we learn and what makes learning more (or less) enjoyable and successful. In this activity participants reflect on and discuss their experience of successful and unsuccessful learning, and things they would like to learn but have not yet tried. In this way they identify important dimensions of the adult learning experience and apply this insight to developing and discussing principles for teaching adult numeracy.

Aims

The aims of this activity are to:

- increase participant's awareness of the factors which contribute to successful and unsuccessful learning;
- identify factors which inhibit or block adults' learning;
- develop principles and strategies for effectively teaching adult numeracy.

Materials and Equipment

- Whiteboard or butchers' paper and pens

Time 40 minutes

Handouts

- 3.1C *Principles of Teaching Adult Numeracy* (from Activity 3.1)
- 3.2A *My Experiences of Learning as an Adult*

Preparation

Photocopy one copy of Handouts 3.1C and 3.2A for each participant.

Conducting the Activity

1. Introduce the activity to participants by referring to the Rationale and Aims or by saying:

   in this activity we are going to examine our own experiences of learning as an adult in order to increase our understanding of what if feels like to be a student in a numeracy class. We will then use this awareness to develop and discuss a set of principles for teaching adult numeracy.

2. Talk briefly about one or two experiences of your own as an adult learner, making sure you include an unsuccessful as well as a successful experience.

   If you can inject some humour, so much the better, as this will warm up the group for the activity which follows.

3. Ask participants to volunteer a list of things they have attempted to learn as an adult. There is no need to record their ideas; the purpose of this part of the activity is to generate ideas and sharing of experiences, (and also to discover what a diverse and talented group of participants you have).

4. Ask participants to spend five to ten minutes completing Handout 3.2A *My Experiences of Learning as an Adult.*

5. When participants have completed the questions ask them to share their responses with others in their group. One way to do this is to take each question in turn, and discuss the similarities and differences in their experiences. Circulate among the groups, listening to the discussion. This will help you pinpoint important issues for the discussion which follows. Allow the discussion to continue for about 15 minutes.

Discussion

The purpose of the discussion is to generalise from participants' experiences some important principles of adult learning.

1. First, draw together the experiences of the group by asking participants to volunteer the factors which contributed to learning, enjoyment of learning and dislike of learning. Record these on butchers' paper under separate headings, for example:

activity	what helped me learn:	I enjoyed:	I disliked:
wind-surfing	- patient 'teacher' - practice! - reading a book on the theory	- making progress - being in the 'great outdoors' - excitement	- falling off + getting wet + cold - feeling incompetent
learning Spanish	- practice - class activities - feedback from teacher - homework	- being treated as an equal by the teacher - class activities and excursions	- big range of skills in the class - moving too slowly at times

Do the same for question 2, which analyses unsuccessful learning experiences, to identify what made learning difficult or frustrating. For question 3, make a list of barriers to learning and strategies for helping a student start, using information and ideas volunteered from participants.

2. The next phase of the discussion makes a connection between participants' findings about learning in general to the learning of numeracy.

Return to the lists you have created and look at them again, this time in relation to the teaching and learning of numeracy.
Select from the lists (e.g. by ticking) the factors which would:

- enhance the learning of numeracy;
- create enjoyment when learning numeracy;
- make learning numeracy difficult, frustrating or unpleasant;
- prevent students from making a start;
- help students make a start.

Encourage participants to offer new ideas as well as the ones from the lists. Ask participants to suggest strategies for counteracting common difficulties or frustrations.

3. Further discussion could follow up these issues in more detail. Questions to ask could include:
    - *Why do you think people who participate in adult literacy courses do not enrol in maths courses?*
    - *How do we make maths more accessible to potential students ?* (e.g. classes at the workplace, classes at night)
    - *What strategies could we use to encourage students to start a maths class?* (e.g. taster sessions, attractive and friendly publicity)
    - *How can we make maths more enjoyable? (*e.g. fun activities)
    - *How can we enhance students' learning of maths?* (e.g. by making it more relevant to needs and interests, using hands-on materials)
4. Distribute Handout 3.1C *Principles for Teaching Adult Numeracy*, and discuss with participants. Questions to ask could include:
    - *How well do these principles fit with you own experiences of learning as an adult?*
    - *Can you suggest further ideas to add to this list?*
5. A final point for discussion:
    - *Do you think an activity similar to this would be useful to your students ?*
    - *If so, what questions might you ask?*

Conclusion

Conclude the discussion by reminding participants that further activities in this kit will provide more detail on how to put these principles into practice. Activity 3.3 *Experiencing Success with Metric Length* is an excellent activity with which to follow this one, as it demonstrates several of these principles.

My Experiences of Learning as an Adult

1. Something I learnt successfully as an adult was

....

....

What helped me learn it was

....

....

What helped me enjoy it was

....

....

Things I disliked about the experience were

....

....

2. Something I failed to learn to my satisfaction was

....

....

What made learning difficult or frustrating was

....

....

3. Something I would like to learn but have not tried is

....

....

What stopped me from trying was

....

....

What would help me start is

....

....

Activity 3.3

Experiencing Success with Metric Length

Rationale

This activity performs two functions; whilst being a valuable lesson for teaching metric estimation, it is also an example of an activity which can be done in class to provide a **successful** mathematics experience and thus boost students' confidence.

Adults who have grown up with the imperial system of measurement often find the metric system both a mystery and an irritation in their lives. Cries of 'I don't understand metrics at all' or 'all that metric stuff means nothing to me' are common, especially among students of numeracy and basic maths classes. People have become quite adept at 'thinking in imperial' and don't imagine that they can change those habits. This presents problems for them when confronted with children learning the newer system or having to talk to tradespeople or shop assistants in the new units.

Some people try desperately to learn to convert, however it seems that habits of trying to convert from one system to another inhibit automatic responses or 'thinking metric'.
In this activity we present some strategies which can be used by teachers for giving students instant success (or at least marked improvement) at estimating or thinking in metric length units.

Aims

To provide an experience which demonstrates to participants how to:

- raise students' confidence and decrease their anxiety by providing them with a successful mathematical experience;
- promote interaction, and break down communication barriers in the classroom;
- teach estimation of metric length.

Materials and Equipment

- Metric tape measures (1 between 2 participants)
- Metric rulers (1 between 2 participants)
- Blank paper (1 piece for each participant)

Handouts

- 3.3A *Estimating Lengths in Metric Units* (one for each participant)

Time 45 minutes - 1 hour

Preparation

Photocopy 1 copy of Handout 3.3A *Estimating Lengths in Metric Units* for each participant.

Conducting the Activity

The 10 Centimetre Measure

Give out a piece of blank paper to all participants.

Ask them to draw, on the paper, without looking at any rulers or tapes, what they **think** is a 10 cm line.

After they have drawn their lines, hand out the rulers, one between two, and ask everyone to measure their line. Ensure that all participants are familiar with which marks represent centimetres on the ruler. (Often this is is not the case for non maths trained people.)

Discuss the accuracy of their calculations with questions like :

- *Who was far too short/too long?*
- *Who was within 1 cm ?*
- *Did anyone have a strategy that they used ?*

If any ideas on strategy are given, make sure they are discussed and explained to the group. This can often lead naturally into the next step, especially if someone has used some part of their hand as a measurement guide.

If no-one has any strategy for the metric measurement you could ask:

- *How do you estimate in inches or using the other system?* (Often the top section of the thumb is suggested).

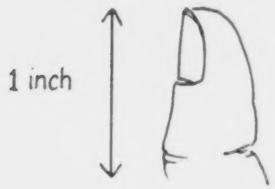
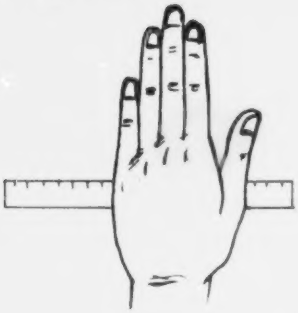


Diagram 1

The next step is to show that we carry around with us very simple aids to estimation.

Still using the example of 10 cm, ask participants to place their hands over a ruler as shown and find a **10 cm reference** Depending on the individual, this is usually across the knuckles or below.

Diagram 2



10 cm. across

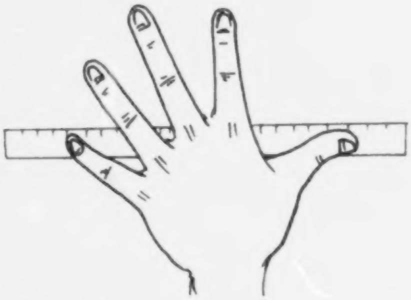
This is a powerful measuring device because it is always with you. (Always on hand so to speak!)

Now move on to a few other useful measuring aids.

The Handspan

Ask everyone to stretch their fingers out as wide as possible - as if they were stretching for a wide chord on the piano - and measure this span.

Diagram 3



20 cm across

Quickly survey the different handspan measurements in the room. Usually these come out to be around 20 cm, with some people a little more and others a little less. Tell the participants that this 20 cm is a fairly common measure for an outstretched handspan and by remembering this, and whether their **own** handspan is **more** or **less**, they will have a very effective **20 cm ruler.**

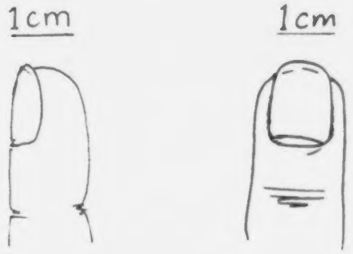
Anecdotes which illustrate the usefulness of this device can be given to put this in context, such as when looking for kitchen fittings for spaces of a given size, clothing for children or frames for pictures.

The One Centimetre Measure

Next, ask participants if they can think of a useful part of their hand for measuring shorter lengths.

Most people have **one finger** which is exactly one centimetre across, either front on or sideways.

Diagram 4



The One Metre Measure

Finally discuss the **one metre** measurement.

Many participants will probably recall buying material by the yard, at the market where it was measured from the nose to the end of the arm.

Diagram 5

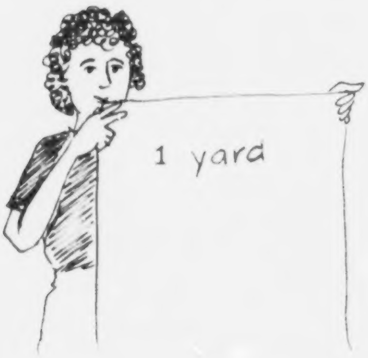
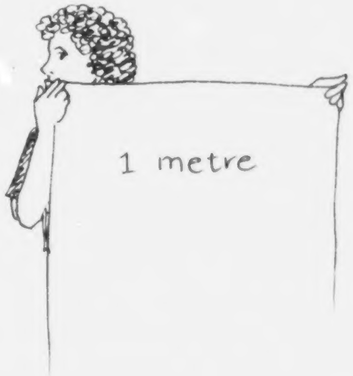


Diagram 6

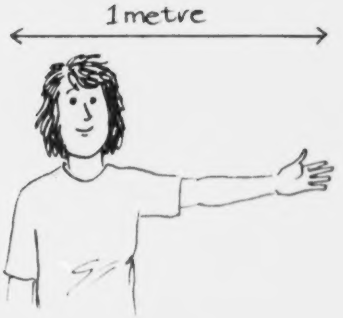


A metre is a little longer than a yard, so the European custom is to turn the face sideways to do the measurement of a metre.

The measurement can also be taken from one shoulder to the end of the other arm when outstretched.

Depending on the length of each participant's arm this will be from the shoulder to the wrist, the knuckles or the fingertips.

Diagram 7

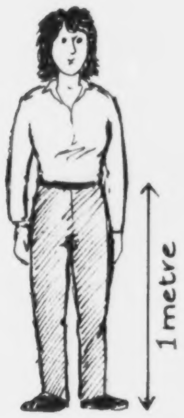


Ask all participants to find their **horizontal metre** this way before moving on to the vertical metre.

The vertical metre is from the floor to the waist, or the hip, or the trouser pocket, depending on one's height. (See Diagram 8)

This is useful for estimating vertical heights such as when buying furniture, since the shoulder-arm measure is not much use in this situation.

Diagram 8



Summary

Quickly go through all the measurements you have covered so far and then distribute Handout 3.3A, one to each participant.

Working in pairs, participants should fill out these worksheets by first **estimating** the lengths indicated using their **body rulers** then **checking** their estimates with rulers and tape measures.

The activities on the worksheet are designed to create maximum interaction which makes this activity an excellent icebreaker.

Closing Activity

Finally ask participants to draw, again without a ruler, a 15 cm line on their page. Compare this approximation with their initial estimation of the 10 cm line and discuss who has improved.

Recommend that participants practise the technique a few times during the next week to consolidate the skills developed.

Final Discussion

A final discussion should centre on applying the activity in participants' own classes. Points to be made are:

- that the activity has almost assured success and is something the students can take home and share with family and friends;
- usually those who were the least proficient at the beginning have showed the greatest improvement and thus it is a model of a good confidence building activity.

Also discuss how the activity can be adapted more simply or extended for particular student groups:

- for example, to simplify, do one measure, such as the ten centimetre across the hand, per session accompanied by lots of practice measuring in centimetres with rulers;
- to extend, practise measuring in millimetres as well as in centimetres;
- measuring one's pace, which is a bit more or a bit less than a metre to estimate larger distances is also another activity to try.

Estimating Lengths in Metric Units

Working in pairs, use the information you have learned to make rough measurements of the following lengths on your partner **without** a ruler or tape measure.

Your partner's	**Rough measure**	**Accurate measure**
- height		
- head circumference		
- foot length		
- distance between eyes		

When you have both made rough measurements (estimations) **check** the actual lengths using rulers and tape measures.

Activity 3.4

Acknowledging Students' Skills and Experience

Rationale

Many students are convinced that they do no have any mathematical skills or knowledge and have little confidence with anything that is labelled as mathematics. However, many of the skills and much of the knowledge they take for granted is actually mathematical, although not recognised as such. This activity is designed to assist students recognise that they know more than they think; that their life experience has provided them with a valuable store of useable mathematical concepts and skills. Acknowledging and validating this knowledge helps to build students' confidence in their ability to further develop their skills.

This activity provides two examples of activities at which students can succeed because of the life experiences they bring to the classroom. It then develops into an exercise where participants, working in pairs or small groups, design a similar activity of their own which is appropriate to their students.

Aims

- To demonstrate models of student activities which acknowledge students' prior experience of mathematical concepts and skills.
- For participants to produce an activity which acknowledges students' existing maths skills and experience.

Materials and Equipment

None

Time 30 minutes

Handouts

- 3.4A *Women in the World*
- 3.4B *Where are the Decimal Points?*

Preparation

Photocopy Handouts 3.4A and 3.4B, one for each participant.

Conducting the Activity

1. Briefly explain the aim of the activity.
2. Distribute Handout 3.4A to half the group and Handout 3.4B to the other half. (Participants can collect a copy of the handout they did not receive at the end of the session.)

3. Divide participants into small groups of three to five, and ask them to follow the instructions on the worksheet, discussing and comparing their results with others in their group. (Allow three to five minutes.)
4. Ask participants to discuss how they would use and extend this activity with their students.

Discussion

Take each activity in turn, starting with Handout 3.4A.

- Ask a spokesperson from one of the groups to read out the completed statement from Handout 3.4A. (The missing words in order are: half, two-thirds, one-tenth, one-hundredth.)
- *What skills are involved in completing this activity?*
- *What are the strengths of this type of activity?* (Responses could include: information provided is from the real world and raises awareness about social issues; can be used as discussion starter and create interest in exploring further concepts and skills.)
- *How could the activity be used and extended in a numeracy class?* Ideas for using and extending Handout 3.4A could include:
  - Asking students if the facts in the passage agree with their experience in the real world.
  - Drawing diagrams which show this information.
  - Writing the information again, starting with 'Men constitute ...'
  - Writing the information again (or drawing a diagram) to show how you think work hours, income and property should be shared.
  - Using it as a basis for further work with fractions.
  - Extending the material into a discussion of decimal-fraction-percentage equivalents.

Now discuss Handout 3.4B, using the above questions to guide the discussion.

- The strengths of this activity include: using a familiar context to introduce and relate metric and decimal concepts; students using their own experience of the world to solve the problem.
- The activity could be extended by students measuring their height in metres and centimetres, discussing relationships between fractions and decimals (e.g. 2.5 km = 1/2 = 2 km 500m); and between different units in the metric system.

Designing a new activity

Now move on to the second part of the activity:

Ask groups to get together and generate ideas for producing a similar type of activity which they could use with their students. (Allow about 15 minutes.)

Then ask the groups to report back their ideas. (If possible, collect the ideas for photocopying and distribution.)

Handout 3.4A

WOMEN IN THE WORLD

Choose a word from the box below to fill in these gaps. Use each word once only.

Women
constitute _______ the world's
population,
perform nearly ________
of its work hours,
receive ________ of the world's income
and own less than ________
of the world's property.

United Nations Report, 1980

one-hundredth half one-tenth two-thirds

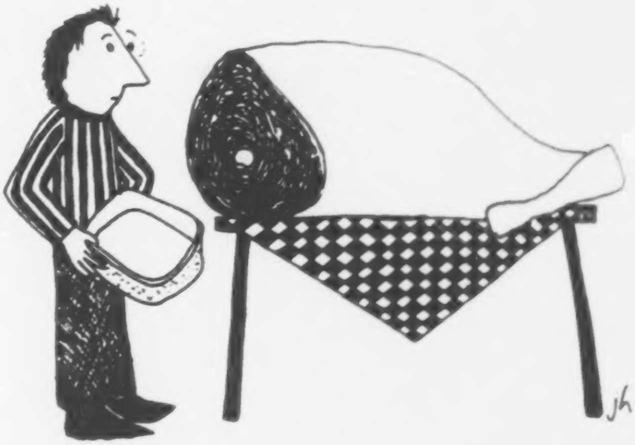
This activity is reproduced with the kind permission of Dilys Norrish and Sue Lord, Petersham College of TAFE, NSW.

Handout 3.4B

Where are the Decimal Points?

Insert decimal points in the most sensible places:

A woman **163 metres** tall gave birth to a baby weighing **32 kilograms.** Before she went into the local hospital **25 kilometres** away (5 minutes drive), she purchased a leg of lamb weighing **25** kg for **$729** and told her husband to cook it at **180°C** for **25 hours.** She also asked him to buy some petrol for their **32 litre** Jaguar, as the price was down to **659 cents** per litre.



Reproduced from *Mathematics:a New Beginning*, B. Marr and S. Helme 1987

Chapter 4. Teaching Strategies.

This chapter examines a range of teaching strategies for meeting the needs of adult numeracy students. These approaches have been selected because they are innovative, and effective with adult learners. By participating in the activities in this chapter, teachers more familiar with traditional methods will have the opportunity to experience and evaluate these approaches for themselves. Teachers new to adult numeracy will benefit from direct experience with a range of strategies. This will enable them to tackle numeracy teaching with advanced confidence and skills. The teaching strategies explored in this chapter include:

- Teaching in Context 162
- Using Hands-on Materials 174
- Interaction and Cooperation 219
- Using Games in Adult Numeracy 259
- Meeting Individual Needs 271

Teaching in Context

Maths classes at school for many adults were confusing, with little or no connection with the reality of their own lives. Skills were often taught in isolation, rules were learnt (perhaps) but certainly not understood and rarely were they applied to situations beyond the immediate demands of textbook or worksheet. A major problem with skills taught in this way is that they do not translate readily to the real life context in which they are meant to be applied:

> *Emphasis on mechanical skills in preference to understanding leads to an inability to define a problem in context. Thus a client may be able to work out the area of a rectangle but not understand how this relates to the quantity of tiles that should be bought to make a splashback for the washbasin at home* (Penny, 1990, p.27).

Australian studies have shown that primary school children unable to solve a problem like 500 - 45, presented as a written problem with the 45 beneath the 500, can easily work out the the change from $5.00 when buying a 45c chocolate bar (Newton, 1988).

Both examples suggest that the formal methods taught in school do not reflect the way people actually think about the numbers they use in real life situations. These methods are more often than not deserted for intuitive or mental calculations.

Literacy teachers already appreciate the value of teaching within real-life contexts. Teaching numeracy can also be approached in the same way, and is most successful when skills are taught, not in isolation, but in the context of people's real needs, interests and goals.

Why Teach in Context?

1. **Adults learn best when the content is personally relevant** to past experience or present concerns[1]. Adults come to class with an enormous amount of collective experience, skills, (many of which they are yet to recognise) needs and goals (both personal and vocational). A program guided by students' own needs and interests will be both meaningful and relevant to them. If the maths taught in class is clearly linked to the situations encountered by students it is much more likely to be put into practice.
2. **Students' motivation and interest is maintained** when the context is relevant and interesting. This produces livelier, more enjoyable classes with students actively involved in the learning process, and more learning as a result.
3. **Students have more control over their learning**. If content is at least partly determined by students' needs and interests, the students can assume greater autonomy and responsibility within the classroom, and a stronger sense of 'ownership' of the topic to be studied.

---

[1] Division of Further Education Vic, (1990) Adult Literacy and Basic Education into the 1990's: a Comprehensive Analysis. Volume 2 .

They can then see for themselves how maths and the real world are connected, as well as the role played by maths in solving problems and making decisions.

4. **Mathematics as a field of study has a history and context of its own.** Teaching mathematical ideas within their own historical context gives meaning, life and richness to mathematics itself and an appreciation that mathematics was created by and for people to solve real problems. Adult students are particularly interested in stories about the history of mathematics and the people who made it.

5. **Empowerment.** Mathematics is a powerful tool for survival. It helps us function effectively in the world and make sense of the information to which we are constantly exposed:

   > *If maths is to be used as a means of empowerment, it must foster skills which develop independence and critical awareness in the student* (Webber, 1988, p.9).

   For the basic numeracy student, independence means having the skills and confidence to estimate how much money is needed to purchase a bus ticket or be able to calculate change from shopping. For an employee, it may mean being able to prepare a quotation at work without having to always ask someone to check it.

   People also need to be able to understand information which is increasingly presented in the media and elsewhere in numerical, tabular or graphical form. Empowerment means the ability to recognise when statistics are being manipulated to support particular viewpoints or the decisions of those in power. Teaching mathematical skills through the examination of information presented in this context gives direct relevance and meaning to these skills and enables students to understand and critically evaluate such information. As a consequence, students are empowered to participate more fully as independent and critical members of society.

6. **Understanding real world issues**. There are many pressing issues confronting our society and the communities within it. Examples are environmental destruction, racism, sexism, health and disease. It can be argued that maths should be one of the vehicles to empower students to understand and take a more active role in finding solutions to such problems (MCTP 1988). In fact, many teachers argue that maths courses should be structured entirely around social issues. Whatever approach is taken it is vital that teachers be responsive and react to issues as they emerge in the media or the local community. If there is an election for instance, it is a perfect opportunity to examine the 'democratic process' as well as build upon students' numeracy skills.

A Note of Caution:

Teaching maths skills within the context in which they are most likely to be used is a highly recommended approach. However, we would like to inject a note of caution, and warn against wholesale use of worksheets or textbooks on 'everyday maths'. Many situations examined in these publications may not interest students at all, nor be relevant to their needs. For instance, it makes little sense for an unemployed student to be given the task of budgeting for an overseas holiday!

If students are working through a series of questions or topics in which they have little or no interest, their involvement in the task will be minimal and they may not make the connection between the skills they are learning and their future applications. It would be more productive to sometimes develop an activity on the spot based on discussions with students than to rely on prepared worksheets from a textbook. Furthermore, not every activity should be based on domestic or 'real life' situations. We also owe our students the opportunity to explore mathematical ideas for their own sake, so the pattern, order and beauty of mathematics can be experienced.

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The Activities

Activity 4.1 ***Maths on the Supermarket Shelves*** uses products from the supermarket as a basis for developing numeracy activities.

Activity 4.2 ***Newspaper Numeracy*** uses newspaper articles to develop the skills of understanding and critically evaluating numerical information in the media which is presented within text and in the form of charts and tables.

You may wish to conclude this series of activities with a discussion of other ways participants have used information, ideas and materials and resources from the real world to enrich their numeracy teaching. This is an excellent way of giving teachers a starting place for developing new ideas.

Activity 4.1

Maths on the Supermarket Shelves

Rationale

The supermarket shelves provide unlimited opportunities to explore and develop mathematical reasoning skills. Information on labels is a rich source of data which can be used to develop the skills which enable students to make appropriate decisions about using the product. This is particularly true for medicines, where it is essential that dosage information is properly understood.

A numeracy activity can be created by bringing products into the classroom. This generates student interest and involvement, giving life to the mathematical skills and knowledge you wish to develop. Metric measurement becomes important and memorable if students practise working out dosages. The ratio and proportion skills involved in mixing powdered milk or infant formula can be discussed with the product at hand and then extended to other situations.

Estimating the number of sandwiches in a loaf of bread, or the number and cost of toilet rolls a family uses in a year is fun to do in the classroom and teaches students important estimation and number operation skills. Having explored these numeracy skills in class with familiar products, students can then be encouraged to practise them in many aspects of their daily lives.

Aims

- To explore the mathematics embedded in familiar supermarket products.
- To provide teachers with the opportunity to use familiar items in developing numeracy activities in small groups.
- To develop sample numeracy activities based on supermarket products.

Materials and Equipment

- Calculators
- Toilet paper (e.g. 6 pack)
- Bottle of cough medicine with medicine glass or teaspoon
- Two or three loaves of bread of different types and sizes
- One packet of dry biscuits or crispbread
- Aspirin tablets or Paracetamol mixture
- One packet or tin of powdered milk
- Whiteboard or butchers' paper and pen.

Handouts

- 4.1A *Questions and Activities Using Supermarket Items*

Time

45 minutes

Preparation

Purchase the supermarket items listed, one item or group of items (e.g. bread) per four participants. As you select the items check that some information is provided on the labels but not too much. It is important that some questions remain to be asked of the students. Write the prices on the items in the shop or straight afterwards from the docket.

Photocopy Handout 4.1A *Questions and Activities using Supermarket Items,* one for each participant.

Conducting the Activity

- A group of sixteen to twenty participants working in four to five groups is ideal for this activity. For smaller groups, you may wish to break participants into groups of two or three so that a range of four to five products can be used.
- Give each group one of the products, and ask the group to brainstorm how they might use the product in a numeracy class. Ask them to generate a list of questions and activities for their product, and record their ideas.
- Stop the discussion after ten to fifteen minutes and ask each group to report back. Encourage other participants to give feedback and make further suggestions. Record these on butchers' paper or a whiteboard.
- Distribute Handout 4.1A which contains discussion questions and activities collected from other groups for using these supermarket items as a numeracy activity. Alternatively, collect lists from this session, add any additional ideas and after the session photocopy and distribute them to participants.
- If you have time, brainstorm a list of questions or activities that could apply to most items, for example, '*Write the price in words*'.

Discussion

- Ask participants to comment on the usefulness of the activity.
- Can you suggest any other good products to analyse?
- Can you think of any other good activities connected with supermarkets? Other household products? [For example: use of junk mail (catalogues) in the classroom for practice with money calculations and estimation; trips to the supermarket to research products; or for some students merely to practise shopping for themselves.]

Questions and Activities using Supermarket Items

Medicine

- How much medicine does a teaspoon hold?
- Measure out dosages for people of different ages using a spoon and/or medicine glass.
- What size dose would a five year old need? An eleven year old ?
- How should the doses be spaced?
- How many doses are in one bottle?
- How much does each dose cost?
- How long will the bottle last?
- What is the cost of a cold?

Toilet Paper

- How much does each roll cost? (In a pack of 4 or 6 rolls)
- What does 2 ply mean?
- How many rolls you would use each week or fortnight?
- How many rolls would you need in a year?
- What would this cost each year? How much money would you save if you didn't use it?
- How many sheets of toilet paper are there in the pack?
- How far would the toilet paper go if you unrolled the whole pack?
- Classes can compare value with other brands looking at length,thickness and price.

Powdered Milk

- How much powdered milk do you need to make one cup of milk? 2 cups? 1 litre?
- Is a litre of powdered milk cheaper or more expensive than fresh milk?
- How would you double the strength?

- What is the percentage of milk fat in the powder?
- What is the use by date? How much time is there until that date?
- Discuss the recipes on the label.

Bread

- Compare different sizes and types of loaves.
- Which one do you prefer and why?
- Which loaf is the better value for money?
- How do the letters A, B, C, D, E on the label relate to the cost?
- How many slices or sandwiches in a loaf?
- Look at the nutritional information. Can you use this to compare the different types of loaves?

Dry Biscuits or Crispbread

- How much is the packet?
- Can you write the price in words?
- What is the weight of the pack?
- How many biscuits in the pack?
- How many biscuits make up 100 g?
- What is a kilojoule? [A kilojoule is 1000 joules, the unit of energy which takes its name from James Joule, a British physicist (1818-1889). An opportunity is provided here to discuss the energy content of different foods, daily energy requirements and other dietary issues.]
- How many kilojoules are there in one biscuit?
- Discuss the relationship between kilojoules and calories (1 calorie is about 4 kilojoules).

Aspirin Tablets or Paracetamol Mixture

- Explain the dose for people of different ages.
- How should the doses be spaced?
- How many adult doses are there in a packet? (or child doses in a bottle?)
- What is the cost of each dose?
- Discuss alternatives to commercial drugs for relieving headaches.

Newspaper Numeracy

Rationale

Newspapers are an ideal source for material on which to base a numeracy activity, with a daily supply of facts and figures, graphs and tables and information about current issues. Newspapers create important links between the classroom and the real world. Using them as a teaching resource enables students to strengthen and apply their numeracy skills in a range of situations. In this activity, participants work in small groups to develop numeracy activities from newspaper articles.

Aims

This activity is designed to generate ideas and strategies for the effective use of newspaper articles in adult numeracy classes.

Materials and Equipment

- Whiteboard or butcher's paper and pens
- Calculators

Time 45 minutes

Handouts

- Handout 4.2A *The Millionaires' Club: a Teacher/Tutor Activity*
- Handout 4.2B *Activities Based on The Millionaires' Club*
- Copies of daily and local papers (one for each group)

Preparation

Collect copies of recent daily newspapers (for example, copies from the past few days) and one or two copies of local papers. Select newspapers which numeracy students are likely to read. Make sure there is at least one newspaper per group.

Photocopy Handouts 4.2A and 4.2B so there are enough copies for each participant.

Conducting the Activity

Introduce the activity, referring to the background notes for this section, *Teaching in Context*, and the rationale for this session.

In the first part of the activity, participants develop a numeracy activity based on the article in Handout 4.2A *The Millionaires' Club*. The participants then go on to develop numeracy activities in small groups based on articles of their own choice.

Part 1: *The Millionaires' Club*

Ask participants to make a group with two or three others. Distribute a calculator to each group and a copy of Handout 4.2A for each participant. Give participants a few minutes to read it, then ask them how they might use the article in a numeracy class.

Discussion questions may include:

- *How would you present the article to students?*
- *How much of the article would you use?* (E.g. only the heading and an enlargement of the table would be appropriate for most groups.)
- *What questions or activities could you develop from this information?*

To stimulate discussion it may be necessary to provide one or two suggestions from Handout 4.2B, but do not distribute it at this stage. It is far more valuable that participants be encouraged to think creatively and develop ideas relevant to their own students.

Allow ten minutes for groups to discuss their ideas. Then ask for two or three suggestions from each group, recording these on whiteboard or butchers' paper. For each idea recorded, ask participants to identify the numeracy skills embedded in the task, question or activity they have invented. Record these on the chart as well.

This will help them focus on the mathematical skills involved and also create awareness of the diversity of skills covered by this sort of activity.

Distribute Handout 4.2B and ask for comments and further suggestions. Remind participants that this is not an exhaustive list, but may help stimulate further ideas.

An issue which may arise here is how to respond to mistakes made in articles. There is a misprint in the item on the table (the price of a Melbourne *Age* in 1955 should have been 0.03, not 0.3). Discuss the pros and cons of correcting mistakes before showing the information to students.

Part 2: Creating a Numeracy Activity from a Newspaper Article

Arrange for each group to select one newspaper from the collection brought to the session.

Ask each group to find an article from the newspaper and develop a numeracy activity from it, writing down questions and activities (these will be circulated later). After ten minutes or so, ask groups to report back their ideas. If there is time, ask for further suggestions during this discussion.

Collect articles and questions from groups. Arrange for photocopying and distribution to participants at a later date.

Conclude the session with a discussion about the strengths and pitfalls of using the newspaper as a numeracy resource.

Handout 4.2A

The Millionaires' Club: a Teacher Tutor Activity

When you join the millionaires' club, don't expect to be rich

By the year 2025, a great many Australians will be millionaires because of the superannuation schemes they now belong to. **SETH GREGORY** examines how much your million dollars will be worth.

Welcome to the YEAR 2000

WHAT A MILLION WILL BUY

Item	1955 $	1990 $	Inflation Factor	2025 $
1 litre milk	.15	.93	6.20	5.77
Loaf of bread	.11	1.25	11.36	14.20
Packet of cigarettes	.30	3.10	10.33	32.03
1 dozen eggs	.55	2.25	4.09	9.20
1 kg steak	.61	9.95	16.31	162.30
Wool trousers	8.00	80.00	10.00	800.00
Inexpensive car	2,100.00	15,000.00	7.14	107,142.86
Return air fare to the UK	1,320.00	2,779.00	2.10	5,850.64
Weekly flat rental	18.00	180.00	10.00	1,800.00
Modest house	11,000.00	140,000	12.72	1,780,800
Toorak Mansion	34,000.00	800,000.00	23.53	18,823,529.41
Average weekly wage*	31.00	423.00	13.64	5771.90
1 litre (quart) whiskey	2.10	26.00	12.38	321.90
Melbourne Age	.3	.50	16.66	8.33

* Wages based on 1954 and 1989 figures

IF YOU are 30 years old, employed and belong to a superannuation scheme, chances are that someday you will be a millionaire. And you won't have to discover gold, invent a new computer or win the lottery.

If your superannuation contributions, and your employer's, total $2000 a year, and if your money is invested in securities paying 12 per cent interest (compounded monthly) or higher, you will have accumulated around $1 million when you reach the normal retirement age of 65.

That's not a bad investment, considering that you and your boss have contributed only $70,000. In fact, even $1200 invested yearly in such a superannuation fund will yield more than $750,000 in 35 years. The amount will be even larger if you are under 30. Eventually, when all Australians are covered by superannuation schemes, we will be a nation of millionaires.

Sounds too good to be true? Well, there are two catches. We may all be millionaires when we retire, but the Government's taxation policy (which penalises lump sum payouts) will force most of us to take our superannuation in the form of a pension rather than a lump sum. So, it will be impossible to spend our superannuation all at once.

Inflation is the second catch. A million dollars is only worth what it can buy, and it is unlikely that by the year 2025 it will buy very much. It already takes 10 million of today's inflated dollars to buy what one million could 35 years ago. Thirty-five years from now, it may require a hundred million.

Now it's always a dangerous business to predict the future and I have no crystal ball. But we can take a tip from the American magazine 'Changing Times', which suggests that one way to tell what the economy will be like in 35 years is to examine what things cost 35 years ago.

The reasoning goes something like this: a cheap pair of kids' school shoes cost £1/10/0, or $3, in 1955. Today, even at K-Mart, the same shoes will set you back $24 or eight times as much. For shoes, then, the inflation factor over the past 35 years is eight. If you assume that inflation over the next 35 years will be similar to the past 35, then school shoes should cost eight times more in 2025 than they do today. In other words, to estimate prices in 2025, we merely calculate price changes over the past 35 years and apply them to the next 35 years. Using this formula, we can estimate that cheap school shoes will cost $192 ($24 x 8) in 2025.

This approach to estimating future prices is not without its problems. For one thing, the calculations assume that the future will pretty much resemble the past and there is no real reason why it should. Nevertheless, it's a pretty good bet that nothing important will change between now and 2025. Over the past 35 years, we have had a variety of governments and inflation rates that varied from nil in some years to 20 per cent in others.

If we assume that we will have a variety of governments and similar swings in inflation over the next 35 years, then the procedure just described should give us a reasonably accurate picture of what that $1 million will buy when we grow old enough to spend it.

The accompanying table lists the 1955 price translated from sterling into dollars, the current price, the inflation factor and the estimated price in 2025 for various consumer items and wages. The 1955 figures come from advertisements in 'The Age', and from the 'Yearbook Australia' published by the Commonwealth Government. The 1990 figures are also based on newspaper ads and government statistics as of June 1990.

In cases where several 1990 prices prevail, the most representative rather than the highest was used. So although they may appear outlandish, the estimates for 2025 are actually quite conservative. For example, there are many places today where men's wool trousers cost more than $80. If I had used a higher 1990 price for this item, say $100, the 2025 estimate would have been $1250 instead of $800. It is also worthwhile noting that although we tend to think of inflation as uniform, some prices increased quite a bit more than others since 1955. International airfares, for example, have increased very little over the past 35 years. Real estate, on the other hand, has exploded.

By examining the entries in the table, it is possible to answer the question of just how far that million-dollar superannuation bonanza will go in 2025 — not very far at all. A moderate house, a cheap car, some new clothes and it will be largely gone. This doesn't mean you should not participate in a superannuation plan. Considering what prices will be like, you may starve to death without one. Just don't expect to be rich.

Source: The Age 25/6/90

Breaking The Maths Barrier, B. Marr and S. Helme, 1991

BEST COPY AVAILABLE

Activities Based on the Newspaper Article: *The Millionaires' Club*

This is not an exhaustive list of ideas but rather a sample of activities which range from those requiring very basic skills to activities which involve more sophisticated concepts and reasoning.

Activities/Questions	Embedded Maths Skills
Introductory discussion about price increases and inflation; student recall of prices from many years ago.	Basic number concepts.
Ask students questions such as 'How much was a dozen eggs in 1955?' 'How much is it predicted to be in 2025?'	Comprehension of numerical information; reading numbers aloud.
Writing the prices of various items in words.	Translating numbers into words.
Ask students to examine table. How close are the 1990 figures to the prices you actually pay for things?	Comparing own experience with information in the media.
Select some items and discuss how the price has changed; students' reactions to these changes.	Comparing quantities; finding price increases by subtraction.
Introduce and discuss the concept of 'inflation factor' as used in the article. (Use examples of wool trousers and rent as the inflation factor is a simple whole number for these items.) How did the writer know that in 1955 a pair of trousers cost $8? How did the writer figure out that trousers would cost $800 in 2025?	Multiplication by 10; logical reasoning.
Describe in your own words what the 'inflation factor' tells us.	Verbalisation of a mathematical operation.

Extend concept of 'inflation factor' to other items. Use calculators to multiply prices by the inflation factor to check 1990 figures and the predictions for 2025. Divide the 1990 figures by the 1955 figures to obtain the inflation factors, and then multiply to obtain prices for 2025.	Using a calculator for multiplication and division, to test a hypothesis and to check calculations.
Which items had the highest inflation factor? The lowest? Can you explain why the inflation factor has not been the same for all products?	Comparing rates of inflation; making inferences.
Ask students in turn to read out each line of information and explain in their own words what has happened to each item. Ask students to round off large numbers to make them more readable.	Verbalising numbers and translating them into meaningful information; rounding numbers.
Look at the figures for the modest house. In two of these the decimal point is missing. Where would you put it?	Comprehension of decimal notation.
If you were a farmer, which sort of farm would you have preferred to have over the last 40 years - poultry, beef or sheep?	Comparing rates of inflation and relating them to the lives of farmers.
If you wanted to predict what petrol or a ticket to the movies would cost in 2025, what information would you need to collect? (Students may like to try and find this information.)	Analysis and research skills.
What assumptions did the writer make in his predictions for price increases between 1990 and 2025? Are these assumptions appropriate?	Critical thinking; proportional reasoning.
Inflation affects some people more than others. Who benefits and who suffers from inflation? How can we protect ourselves from the effects of inflation?	Critical awareness of social issues.

Using Hands-on Materials

Hands-on experience is fundamental to all learning, and the learning of mathematics is no exception. Using hands-on materials in teaching maths to adults enriches the learning experience by encouraging visualisation and expanding the range of senses involved to include touch, sight and movement.

By manipulating concrete objects, students learn by exploration and discovery, becoming active participants in the learning process.

Hands-on materials are also useful as an aid in problem solving. Using pieces of paper, counters or objects they can move around encourages students to visualise the problem and take risks in trying out solutions.

Most adults, however, were taught maths using traditional paper and pencil methods, and were expected to memorise rules and formulas without really understanding them. A classic example is the moral argument of 'borrow and pay back' for subtraction, which tends to mystify other than explain this basic arithmetical process.

Concrete materials are a powerful means enabling students to understand that such rules (albeit expressed in terms of place value and regrouping) do make logical sense.

British psychologist Richard Skemp (1986) warns against attempts to learn mathematics by rote, concluding that if basic concepts are not properly understood, later learning which builds on these concepts becomes an unnecessarily difficult, if not impossible task. Perhaps this is the reason that most adult students have little real understanding of maths and consider it to be a difficult subject.

Many teachers involved in adult education were the victims of similar educational methods as their students. Having learnt maths using traditional paper and pencil methods they may not feel very confident with alternative approaches. Teachers using hands-on materials for the first time may also fear the reaction from students when they produce materials which they may think belong in the kindergarten!

But it is important to overcome these initial reservations. Once you have used them and seen how effective they are, hands-on materials will become a valuable part of your teaching repertoire.

Using Hands-on Materials for Learning Mathematics

This approach to teaching mathematics owes its origins to the work of Piaget and Dienes. They argued that before a child acquires the ability to manipulate and abstract concepts mentally, he or she must pass through a stage of concretely manipulating objects. We believe that this is true also for adults: **no matter how old the student is, no verbal or written instruction can replace what can be learned through experience.**

Hands-on activities can give students the chance to experiment with many concepts, such as number, place value and fractions. In doing so they begin to conceptualise relationships and make the connections that are necessary before proceeding to more formal work. Thus, learning is not simply the transfer of knowledge from teacher to student, but is an **active** process:

> *Learners construct their own understanding of the world: their constructions are personal and necessarily linked to their own experiences. When the mind experiences something new, older perceptions and concepts are pushed around and shaken up so that the new information can be fitted in with what is already known.... Learning is a natural process, a process of making sense of things* (Mathematics Frameworks 1988, p.22).

A four stage learning model

How can we explore this process in more detail, and what implications does it have for teaching adult numeracy? Working in Footscray, Victoria, with students from a non-English speaking background, Ballagh and Moore (1987) developed a framework for mathematics learning which incorporates hands-on experience. This model derives from research into experiential learning which draws together the ideas of Dewey, Lewin and Piaget (Kolb, 1984).

The process of learning maths can be understood as a four stage cycle which involves four learning modes: concrete experience, reflection, abstraction and application.

Stage 1: Concrete Experience

In this stage new concepts are presented in a concrete way through a practical activity or physical model. This gives students the opportunity to draw on their prior knowledge and experience together with the sensory information gained in the activity, to form an intuitive understanding of the concept.

It may also serve a motivational purpose by engaging students in an enjoyable and non threatening activity. In exploring fractions for example, students may start by collecting pieces together to make circles. They may name pieces they already know, and name unfamiliar sizes according to how many pieces make a circle. They may notice that two pinks (quarters) fit exactly on top of a red (half), which leads us to Stage 2.

Stage 2: Reflection

Students naturally use language to explore and reflect upon the information they are gaining at a concrete level. The role of the teacher is to encourage students to describe, discuss, and explain the relationships they are observing. It is important that they start by using informal language, expressing what they have observed in their own words. For example, students may express their observations initially by commenting that two pink pieces are the same as a red piece. The teacher then begins to encourage the use of more formal language and may introduce some new words or notation, such as the words numerator and denominator or the idea of equivalent fractions. This process of reflection allows students to build on their experience, discover important relationships for themselves, articulate these relationships and begin to formalise them in writing.

It is during this process of examining concepts anew that students may discover that rules or assumptions they made in the past do not work any more. For instance many students assume that when adding the fractions 3/8 and 1/4 you simply add the top and the bottom separately to give 4/12. If students have the opportunity to work with fraction circles, they would find that this rule is inconsistent with what their experience tells them, because combining the fraction pieces would make a piece much bigger than 4/12 . If students continue exploring the problem and replace the 1/4 with 2/8, they would then have in front of them the fraction 5/8, the result of adding 2/8 and 3/8. When students articulate a new, and more appropriate explanation based on their own experience, they are developing real understanding and not mindlessly applying rules. The importance of written and spoken language at this stage cannot be over- emphasised and useful strategies are explored in more detail in chapter 5: *Language and Mathematics*.

Stage 3: Abstraction

Once students have discovered important relationships, they are ready to integrate their observations into logically sound procedures or rules, translating their own language into formal language and symbols. (For example, *To multiply two fractions you multiply the top numbers together and the bottom numbers together.*) It is important that students have access to the hands-on materials in order to check their predictions. For example having predicted that 3/8 of a winnings is more than 1/4 of it , using their rule for equivalent fractions (i.e. 1/4 = 2/8, which is less than 3/8), they can return to the kit to check their results.

Stage 4: Application

The final stage in this process is applying their understanding of new concepts in unfamiliar situations, for example, applying their knowledge of equivalent fractions to finding percentages. Students able to generalise to fractions which are not in the kit can convert 25/100 into 1/4 and thus solve a problem using a 25% discount.

(We recommend that equivalent fraction skills are used only for percentages which convert easily to simple fractions. For working with more complex percentages we recommend using a calculator. For details see Marr and Helme (1987), *Mathematics: A New Beginning*.)

Another model of learning mathematics

British educator Pamela Liebeck, in her book *How Children Learn Mathematics* (1984) proposes a similar model for tracing the progress from concrete experience to abstract conceptualisation in mathematics.

She categorises the sequence into the following four stage process:

E EXPERIENCE with physical objects.

L LANGUAGE that describes that experience.

P PICTURES that represent the experience.

S Written SYMBOLS that generalise the experience.

Her model is similar to that proposed above except for the inclusion of pictures (or diagrams) and the exclusion of the application stage.

The addition of pictures, diagrams, graphs, charts and other visual aids is a useful one, and underlines the importance of visualisation in learning mathematics. For students learning about fractions, diagrams may help students visualise the concept they are learning without the need to resort to hands-on materials. Diagrams of fractions are common on many worksheets and provide useful reinforcement of fractions concepts such as relative size and equivalence.

However, when students are drawing their own diagrams to help them solve fraction problems, take care that these are accurate enough to be helpful. Messy and inaccurate diagrams can be very misleading and create confusion in the learner.

Although Liebeck does not take her model through to the application stage, she points out elsewhere that 'to understand mathematics we need to explore the real problems' (p.12), recognising that the ability to put one's ideas into practice is the test of real understanding.

Using Hands-on Materials for Problem Solving

In teaching numeracy we aim to do more than teach students how to do standard 'sums'. Students need to develop the skills and confidence to apply what they have learned in new and unfamiliar situations. For instance, a student may be faced with a real-life decision such as which type of loan to take out, whether they can afford a car, or figuring out how much timber to buy for a project at home.

These sorts of problems do not have standard solutions, so many people 'freeze up', unable to apply what they know in this new situation. Teaching problem solving techniques gives students the resources and strategies for overcoming this block and tackling non routine problems with more confidence.

(Problem Solving is discussed in more detail in chapter 2: *What do we Teach?*)

Using hands-on materials is one of a range of problem solving strategies but one which we wish to emphasise for beginners because it provides a powerful means of getting the student involved with the problem, both visually and physically. As one student commented:

> *Using the materials got the problem out of my head and onto the table, where I could* **see** *what to do with it.*

Pieces of paper, cards or objects which can be moved around encourage students to take risks. In all the activities in this section, participants use hands - on materials to solve problems. For instance, in Activity 4.4 *Problem Solving Stations,* when solving the problem *Cats and Budgies,* participants actually make models of the animals using cards and toothpicks, and experiment until they reach the correct number of heads and legs. In another problem,*Working out the Roster* they are required to place nametags in a timetable and move them around until all the conditions in the problem are met.

When students see the value in taking risks and trying out their own ideas they are developing important skills for learning and problem solving in any situation.

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The Activities

Three activities have been chosen to demonstrate the value of hands-on materials, both for developing an understanding of mathematical concepts and as an aid in problem solving.

Activity 4.3 ***Modelling Place Value, Addition and Subtraction***, explores the use of a physical model for understanding our base ten number system. It also examines the processes of carrying in addition and subtraction.

Activity 4.4 ***Problem Solving Stations*** explores the use of hands-on materials as an aid in problem solving. A collection of problems are placed in various locations around the room and participants are encouraged to solve the problems using the hands-on materials provided.

Activity 4.5 ***Experiencing Fractions*** explores the use of hands-on materials in developing understanding of basic fraction concepts.

Activity 4.3

Modelling Place Value, Addition and Subtraction

Rationale

This activity provides an example of the use of hands on materials to model basic operations. At the same time it allows participants to consolidate their own understanding of the base ten number system; and the process of carrying for addition, and decomposition for subtraction.

Aims

- To use drinking straws to model numbers and develop the concept of place value.
- To illustrate and clarify the the algorithms for addition and subtraction.
- To facilitate discussion of our base ten number system.
- To facilitate discussion of appropriate methods for subtraction in a variety of situations.

Materials & Equipment

- Drinking straws cut in half (100 whole straws per group of 4)
- Rubber bands (1 packet)
- Manilla folder (1 per pair)
- Coloured card or paper
- Dice (2 per group)

Handouts

- 4.3A *Investigating Addition and Subtraction Using Straw Models*

Time

45 minutes to 1 hour

Preparation

Read the background notes; *What Number Skills are Most Important* in chapter 2 (p.76)

Make up place value kits. Each consists of straws, number cards and a place value board assembled as described below.

Straws

For each group of 4 participants:

- Cut the straws in half (easy with scissors).
- Bundle about 180 together in separate groups of 10 using a rubber band to hold them in a bundle. Leave the remaining straws loose.

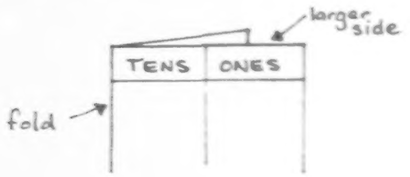
- Take ten of these bundles and band them together to form a cluster of 100 (10 lots of 10).

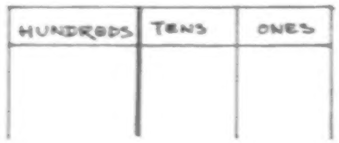
Number cards

- Cut the coloured card into pieces roughly 4 cm by 4 cm.
- Write one digit on each card making altogether 3 copies of each of the digits 0 to 9 for each group.

Playing Board

- Make 1 per pair. Use a manilla folder divided as shown, with heavy coloured lines.





Conducting the Activity

Groups of 4 are the most suitable for this activity. At times they can be broken into two pairs.

Before beginning the activity with the materials it is useful to encourage participants to reflect on the structure of the base ten number system.

Ask them to spend a few minutes imagining they are in a situation where they don't know the numbers of this language and are using their fingers as a counting and recording device, e.g. counting sheep coming out of a run, or people entering a shop.

Possible questions you can ask:

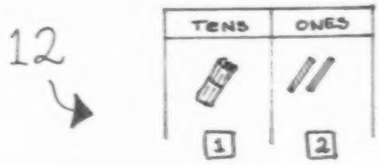
- *What happens when you get past ten?*
- *How do you keep a tally?*

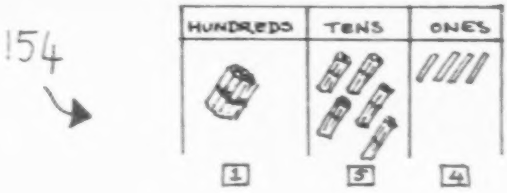
  [The point being made is the need to make some kind of mark or record each time the 10 is reached, so that the fingers can be used again.]

Part 1: Exploring the Materials

Hand out a kit to each group of 4 participants and ask them to examine its contents. [You want them to realise that there are singles, bundles of ten, and groups of ten lots of ten. Also there are cards to represent single digits, spare rubber bands, and a board divided into columns: ones, tens, and hundreds.]

Now briefly allow participants time to model some numbers with the straws and represent the numbers with the digit cards underneath as shown.





Compare this to methods used in the finger counting situation to represent reaching ten.

Once ten straws appear in the last column they must be bundled into a **ten** and carried to the tens column.

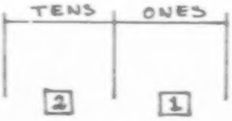
Why ... ? Or *How do you make sense of this with your students?*

Aside from the comparisons to counting on your fingers the analogy of an electronic device could be used, e.g. a digital clock, meter, or an odometer in a car where there is a space for each digit. Digits go only as far as **nine** then we move to a **one** in the next space and start again.

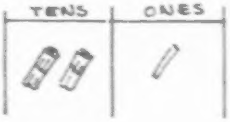
Note: Using this model the single straws can be physically made into a bundle of ten and actually **carried** to the next column. Other base 10 materials replace 10 singles with a different object, e.g. when using MAB blocks 10 blocks are replaced by a long piece. Stress that **only bundles of 10 are allowed in the tens** column.

When using this model with students you would want them to spend some time modelling numbers, saying them out loud, and placing corresponding digits in their correct positions. For example

teacher says: *Model 'twenty one'*
student: puts out digits



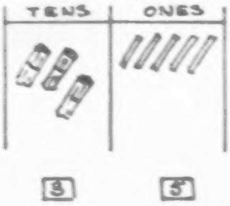
and the corresponding straws



alternately

teacher: models a number

student: places digits
and says aloud 'thirty five'



Part 2: The game 36

To introduce the idea of replacing 10 singles with a bundle and encourage practice with the materials we have introduced a simple game. A game provides motivation for continued repetition.

The group of 4 should divide into two teams of two for this part of the activity.

Each **team** needs a game board

Each **group** needs two dice.

One member of the team rolls the dice and the other member places straws on board according to the sum thrown.

After each throw the new straws are added to those already on the playing board. The winning pair is the one which reaches a total of 36 straws on the board or as close as possible to this target without going over.

Rules

1. No more than 9 single straws can be placed on the 'ones' column.
2. If you roll doubles you have that score and you can have another turn as well.
3. If you reach 30 or more you may choose to 'sit' or 'pass' (as in 21 or pontoon) and hope that your opponents do not get closer to 36 than you are.
4. Players can choose to count both or one die from the two thrown.

Note: Encourage participants to verbalise what they are doing, as they would like their students to do. For example:

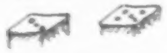
> *I'll take both numbers on the dice. That makes 7 ... I add 7 straws to the ones, which gives me twelve... I have to make these into a group of ten and put them in the tens column.*

An example of play is included to clarify these rules.

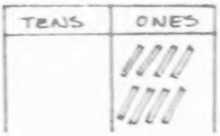
Example of Play

Team A Play first

1st turn



- Roll 3 and 5 on two dice.
- Put 8 singles on the board.



- Hand dice to B team.

Team B Play second

1st turn

- Roll double 6.
- Put 12 straws on the board:
  1 bundle of 10
  2 singles.
- Play again because of the double 6.
- Roll 3 and 5.
- Put 8 more on the board.
- Notice 10 in the ones column and change them for a bundle of 10 which is carried into the tens column.
- Give dice to A team.

2nd turn
- Roll double 5.
- Put 1 bundle of 10 on board.
- Roll again (because it was a double).
- Roll 3 and 6.
- Put 9 single straws on board.
- Notice too many in ones column.
- Make a bundle of ten and carry it to the tens column.

3rd turn
- Roll 4 and 3.
- Put 7 in the ones column.
- Change 10 singles for a bundle.
- Notice how close they are to 36.
- Decide to sit on 34.

2nd turn
- Roll 5 and 6.
- Put 10 bundle and 1 single.

3rd turn
- Notice team A have 34.
- Roll the two dice giving 2 and 5.
- They take the 2 only because 6 would put them over 36.

4th turn
- They must better 34.
- Roll 4 and 5.
- Either would put them out so they concede defeat.

Team A wins.

Part 3: Investigating Addition and Subtraction

Participants should work in pairs for this activity.

- Give each pair a copy of the handout *Investigating Addition and Subtraction Using Straw Models* (one sheet between them should encourage cooperation).
- Circulate amongst the group listening to the language being used and the methods of writing in the symbols they are advocating, for discussion later.

Discussion points

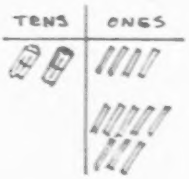
- The parallel of **carrying the bundle of ten** into column compares with the familiar language of the algorithm

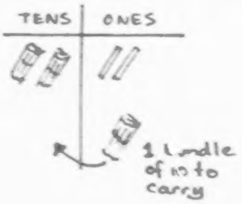
e.g.

$$\begin{array}{rr} & 24 \\ + & \textcircled{1}\;\;8 \\ \hline & 2 \\ \hline \end{array}$$

In the calculation

Eight and four gives me 12
I write the two and carry one.





Using the model

Eight and four in the ones gives me one bundle of ten to carry to the tens column and two singles left.

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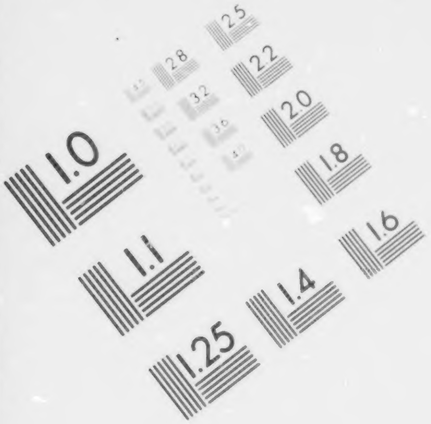
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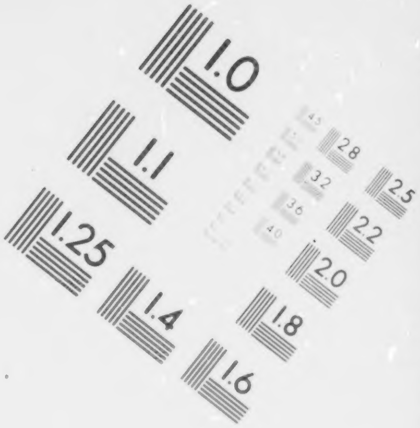
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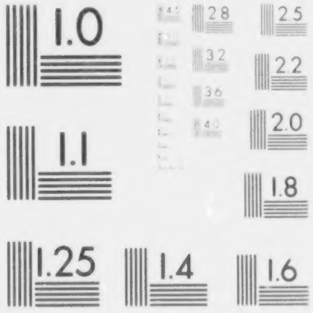
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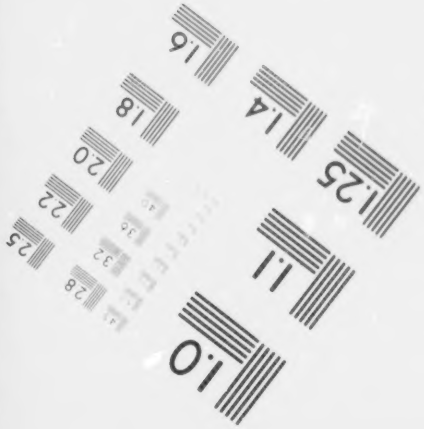
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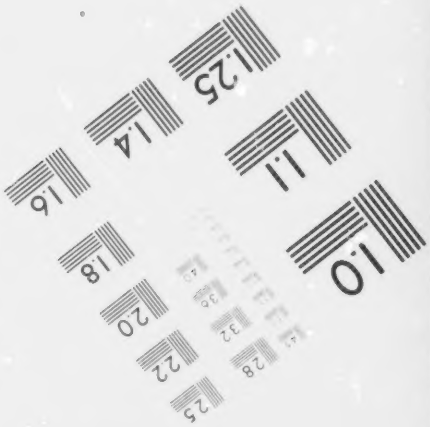
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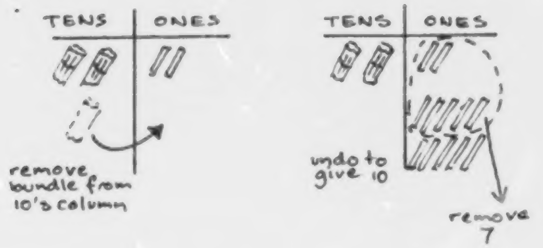
- **For decomposing**

eg $\begin{array}{r} 3\,{}^{1}2 \\ -\ 7 \\ \hline 5 \end{array}$


TENS
ONES
remove bundle from 10's column
TENS
ONES
undo to give 10
remove 7

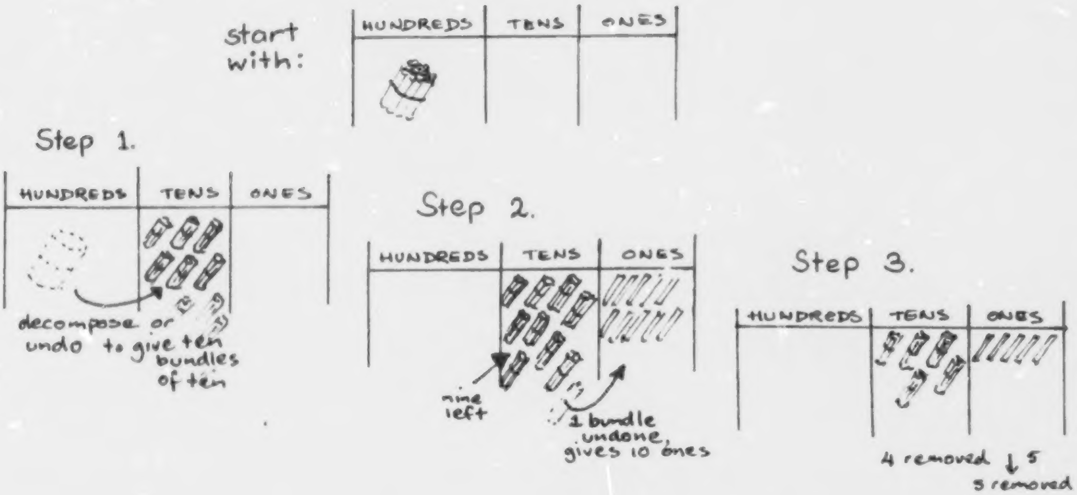
Using the model

One bundle of ten has to be taken from the tens column, undone to create ten singles then seven can be removed.

Various different methods of setting out suggested by participants may need to be discussed by the group. Allow them point out the positive and negative features of each.

- **Use of the model for 100-45**

  If participants attempted this problem with the straws they would have gone through the tricky steps of decomposing a hundred cluster into bundles of ten, moving 4 bundles into the tens column, and then decomposing one of those bundles in order to remove the five singles.


start with:
HUNDREDS
TENS
ONES
Step 1.
HUNDREDS
TENS
ONES
decompose or undo to give ten bundles of ten
Step 2.
HUNDREDS
TENS
ONES
nine left
1 bundle undone, gives 10 ones
Step 3.
HUNDREDS
TENS
ONES
4 removed
5
5 removed

writing this is particularly messy:

$$\begin{array}{r} 100 \\ -45 \\ \hline \end{array} \rightarrow \begin{array}{r} \overset{10}{\cancel{1}}00 \\ -45 \\ \hline \end{array} \rightarrow \begin{array}{r} \cancel{1}\overset{\cancel{10}\,9}{0}\overset{10}{\cancel{0}} \\ -45 \\ \hline 55 \end{array}$$

- **Alternative Methods**

  It is however far better that our numeracy students be looking for shorter methods wherever possible. Here, counting on to 50 then 100 would surely be more efficient than the algorithmic or formal method. For example:

I need 5 more to give 50

then .... 50 more to give 100 .... total ... 55

This question was included to begin such a discussion with your group. The teachers and tutors should be encouraged to think about methods other than the formal in situations such as this where they would be more beneficial These correspond to many everyday situations such as: *How much change if I give $1, $10?* Practice at giving change or counting on in this method is valuable for numeracy students.

Mention also estimation is a useful skill to be teaching students when looking at problems such as this. Encourage discussion of other quick subtraction methods.

Questions You May be Asked

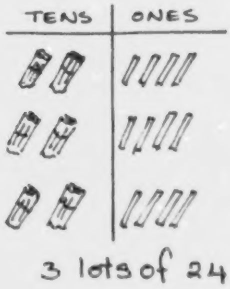
Often questions emerge at the end of this activity which are difficult to answer off the top of one's head. One may involve multiplication using straws:

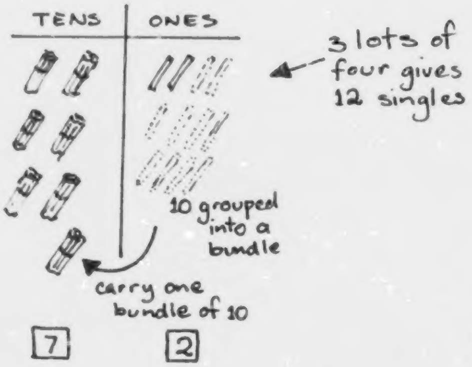
Can you use the straws to model multiplication?

Ideas for response

The algorithm for something like 3 x 24 can be modelled as long as the concept work has come first: this means 3 lots of 24.

Modelling





Obviously for higher numbers the materials become cumbersome and after the initial modelling of one or two calculations students may be encouraged to imagine, or draw diagrams of, the situations. To lead up to the 25 x 35 type of problem, students might visualise and hence generate rules for

20 x 3
then 20 x 34
and finally 25 x 34

Another common enquiry is:

How does the old method of subtraction work?

e.g. 52-18 :

$$\begin{array}{r} 5\overset{1}{2} \\ -1_{1}8 \\ \hline 3\;4 \\ \hline \end{array}$$

borrow one to make 12

and pay back one below

Ideas for Response

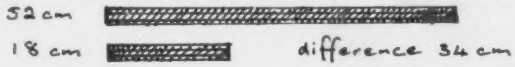
This method didn't ever come with an explanation in the good old days - perhaps just the morality of paying back after borrowing was sufficient argument.

What essentially happens is that 10 is added to both of the numbers

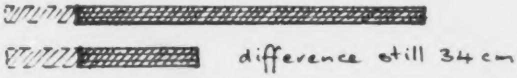
$5\overset{1}{2}$	← add 10	giving 50 plus 12 →	62
$1_{1}8$	← add 10	giving 20 plus 8 →	28
34		difference 30 plus 4	34

The **difference** between the two numbers is kept the same. A visual method of thinking of this is explained in the following:

Imagine two pieces of wood exactly 34 cms different in length.



If you add 10 cm to each of them the difference will still be 34.



Handout 4.3B

Investigating Addition and Subtraction Using Straw Models

An exploration activity for teachers and tutors working in pairs.

Note: this is a teacher exercise to allow a quick exploration of the concepts. It is not a student activity sheet

For each of the following:

a) model the 'sum' or calculation, with straws and go through the motions required to reach an answer;

b) talk through the process using the language you would like your students to develop;

c) explain to your partner how you would translate the operation with the straws into a written calculation using figures.

Addition

1. 23 + 5	Add five straws.
2. 24 + 8	What do you do with the extra straws in the ones column?
3. 35 + 12	
4. 37 + 18	
5. 65 + 3	Do this if you have enough straws.

Subtraction

6. 38 - 3	Remove 3 straws.
7. 32 - 7	How do you remove 7 singles when there are only 2 straws in the ones column?
8. 43 - 16	
9. 100 - 45	Is this model the best way to do such a problem?

Activity 4.4 Problem Solving Stations

Rationale

Stations are maths activities which are placed in various locations around the room. People work on them by moving around the room, preferably in pairs to facilitate discussion, taking as much time as they need to think through and solve the problems. They are an excellent means of providing a 'moving start', giving participants the opportunity for interaction with each other and involvement with mathematics as soon as they arrive. This helps solve the problem of coping with early birds and latecomers, and alleviates the awkwardness of people sitting and waiting for the program to start.

Another valuable feature of stations is that they can be made available throughout the program for people to work on and discuss during meal and tea breaks. Participants are then free to return more than once to problems they have found difficult or challenging, and will often do so after discussion with someone else.

Stations are also an innovative and non-threatening way to present problem solving to students, particularly spatial problems and problems requiring logical reasoning.

They can be used in a number of ways: as a 'moving start', as a class activity or to provide a problem solving activity for students who finish tasks and have time for further work. Stations allow students to work on problems without the pressure of getting an answer immediately, and allow time for experimentation and risk taking.

Students working together can reach a solution through discussion and manipulation of the hands on materials which are usually provided with the problem. Students may also work alone, with as much time as they need to think through a problem, strengthening their problem solving strategies and building their confidence.

Once students have experienced success with these problem solving stations, new ones of increasing difficulty can be presented to them.

Aims

The aims of this activity are for participants to:

- appreciate the value and use of problem solving stations as a student activity;
- explore and develop problem solving strategies, particularly the use of hands on materials;
- develop and practise the skills of working in pairs to solve problems.

Materials and Equipment

- Scissors
- Blu-tac
- Glue

Time 15 to 20 minutes for problems, 10 minutes for discussion

Handouts

- 4.4A *Problem Solving in Pairs*
- 4.4B *Problem Solving Stations*
- 4.4C *Answers to Problem Solving Stations*

Preparation

Read the background notes on problem solving in chapter 2. This will prepare you for later discussion of the strategies used to solve the problems in this activity. Photocopy the six problems provided in Handout 4.4A, including enough copies for participants to take away with them. Full instructions are provided on the handout on how to present the materials. Set up the problems before the workshop begins, at locations around the perimeter of the room. If you are expecting more than fifteen participants, make a second set of stations to allow everyone easy access. You may also like to expand the content by including the problems from the collection of student problem solving activities in chapter 2 (Handout 2.8B). Problems from any other source can also be adapted as stations.

Photocopy one copy of Handouts 4.4A *Problem Solving in Pairs* and Handout 4.4C *Answers to Problem Solving Stations* for each participant.

Conducting the Activity

The activity is best done by individuals or in pairs.

Time can be allocated for this activity within the workshop time adjacent to meal and tea breaks or, preferably, before a session begins to provide a 'moving start' which caters for early and late arrivals.

Move around the room and talk to participants, encouraging them to attempt the problems (sometimes you have to work at this). Suggest they team up with someone else. Working in pairs and and helping each other is a valuable way to develop problem solving skills. Encouraging someone else also teaches us how to develop these skills in our students. Some people like to assign roles of solver and listener and alternate these roles. Using this approach, the solver talks aloud while they solve the problem and the listener encourages them to verbalise, asks for clarification and checks for accuracy. Distribute Handout 4.4A as you move around the room. Encourage participants to make use of it as they solve the problems.

Ask participants to:

- Discuss with their partner the problem solving strategies they are using to solve the problem. (Talking over the problems with a partner has been found a useful and enjoyable way to succeed in this exercise).
- Practise talking maths and helping each other.

- Use questioning strategies. If they see a solution quickly, ask leading questions that will help their partner solve the problem. If they are having trouble seeing the solution themselves, what questions can they ask that will help them solve the problem?
- Think about the way they can use stations in their classroom. (Remind participants that copies of stations will be made available.)

Bring the group together for discussion.

Discussion Questions

Ask participants for comments on the activities.

- *Which ones did you enjoy? ... find challenging?... dislike?*
- *Did the hands-on materials help you solve the problems? How?*
- *What problem solving strategies did you use?* (e.g. grids, looking for patterns, guess and check, drawing a diagram, making lists or charts)
- *What maths concepts or skills did you learn from solving those problems ?*

Conclusion

Distribute Handout 4.4 *Problem Solving Stations* and Handout 4.4C *Answers to Problem Solving Stations*.

Problem Solving in Pairs

Tips for working in pairs (and questions you can ask.)

1. Listen carefully to your partner:

- *Can you repeat that ?*
- *Slow down, I'm not following you.*

2. Encourage vocalisation:

- *What are you thinking?*
- *Can you explain what you are doing?*

3. Ask for Clarification

- *What do you mean ?*
- *Can you say more about that ?*

4. Check for Accuracy

- *Are you sure about that?*
- *That doesn't sound right to me.*

Tips for the listener (if playing roles of solver and listener):

DO NOT solve the problem yourself.

DO NOT tell the solver how to solve it.

Problem Solving Stations: Materials and Instructions

There are six problem solving stations provided. We recommend that problems are attached to the wall with Blu-tac and that the materials, diagrams and grids are placed on tables positioned against the wall underneath the problem. If possible, photocopy all materials onto coloured card or mount onto card.

Problem	Materials and Instructions
Cats and Budgies	About 50 toothpicks or matches for legs. Beans, counters or pieces of card for bodies.
Round in Circles	7 counters or pieces of card, with the numbers from 1 to 7 printed on them. The diagram mounted on card.
Working out the Roster	Mount roster timetable and nametags on card, cut out nametags.
Squaring the Triangle	Mount on card and cut out the shapes provided.
Jumbled Jobs	Mount grid and ticks and crosses on card. Cut out ticks and crosses.
Colour Logic	Mount grid on card. Cut out 6 rectangular pieces of card to fit on the grid, one each of yellow, green, brown, black, red and blue.

Hints for storing:

Store the set of stations in a folder or large envelope, and store the hands-on materials in smaller, labelled envelopes.

200

Cats and Budgies

Gina and Tom have cats and budgies.

- They counted the heads and got 10.
- They counted the feet and got 34.

How many cats and budgies do they have?

Use the hands-on materials to help you solve this problem.

Breaking The Maths Barrier, B. Marr and S. Helme, 1991

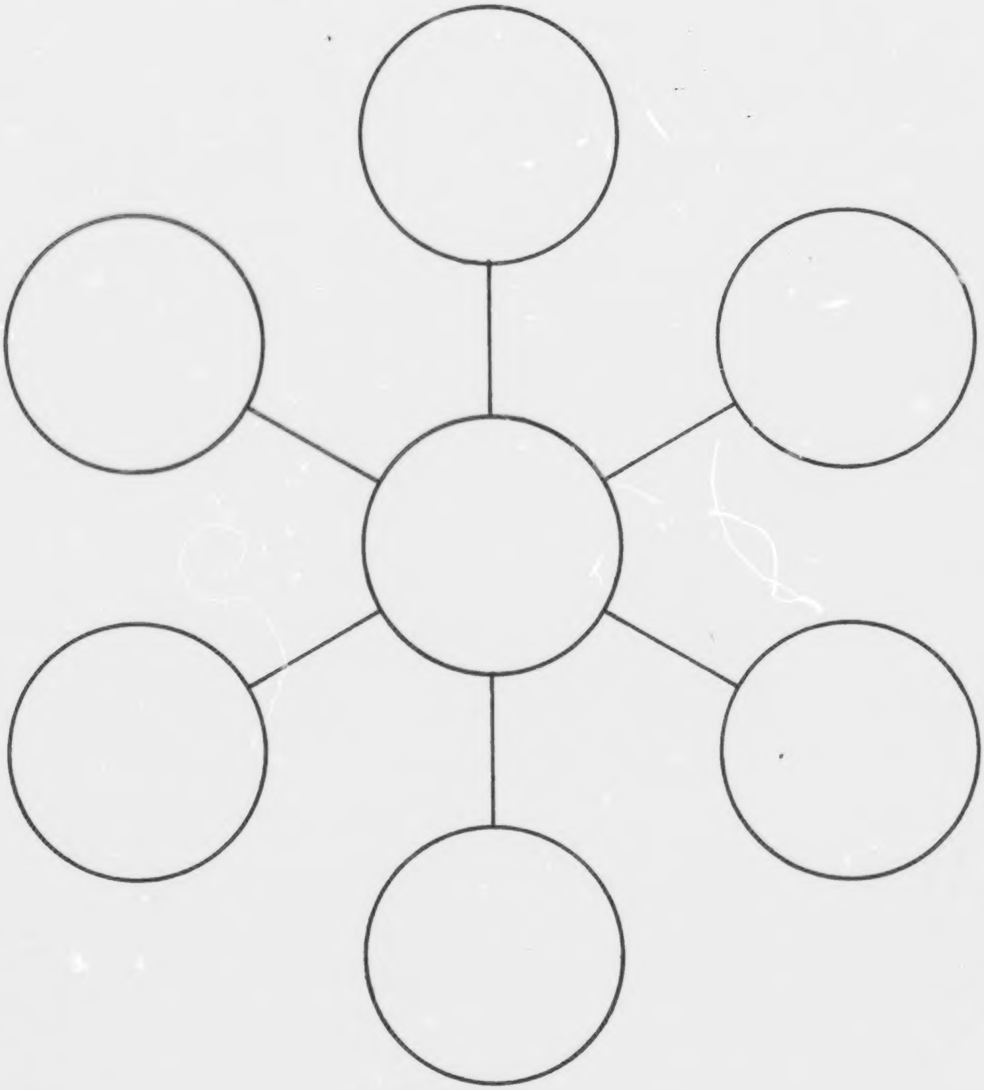
Round in Circles

Using the numbers 1 to 7, fill in the circles below so that each row of numbers adds up to 10. Each number can be used only **once.**

Use the hands-on materials to help you.

203

Round in Circles



Handout 4.4B p.5

Working Out the Roster

Vera is working out the duty roster for the school holiday program. Each person works two half days on the program. There are five staff - Joan, Bridget, Cathy, Pat and Frank. Each person has other commitments which limit their availability such as childcare, work and part time study. Vera makes a list of their requirements as follows:

- Joan can't work Monday, Thursday or Friday.
- Bridget wants to work all day Thursday.
- Frank wants to include Monday morning in his roster.
- Joan prefers two half days to a whole day roster.
- Pat wants to work two afternoons in a row.
- Frank and Cathy can both work on Friday.
- Cathy can only work in the mornings.

Vera manages to complete the roster to everyone's satisfaction. Can you? Use the grid and nametags to help you.

Breaking The Maths Barrier, B. Marr and S. Helme, 1991

Handout 4.4B p.6

Working out the Roster

	Monday	Tuesday	Wednesday	Thursday	Friday
a.m					
p.m					

Frank	Cathy	Joan	Bridget	Pat
Frank	Cathy	Joan	Bridget	Pat

Cut out nametags and place on the table next to the roster.

Breaking The Maths Barrier, B. Marr and S. Helme, 1991

Squaring the Triangle

Can you arrange the four pieces to make a square?

Or an equilateral triangle (a triangle with all 3 sides and angles equal)?

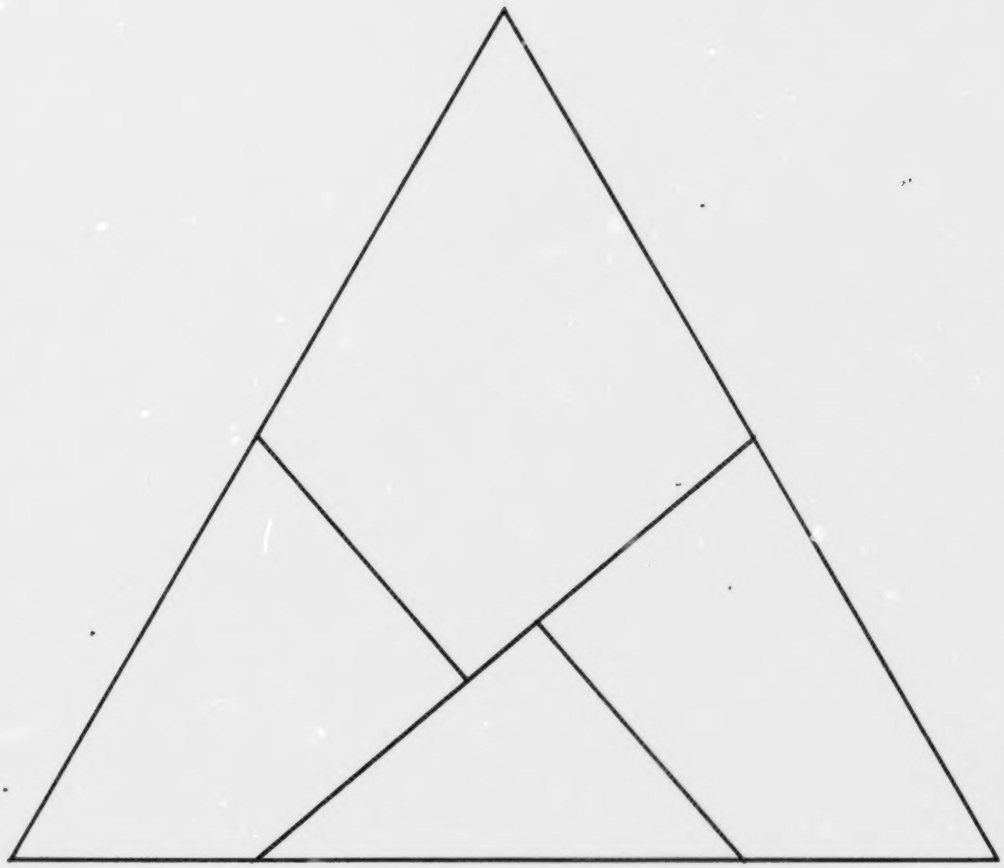
209

210

Handout 4.4B p.7

Breaking The Maths Barrier, B. Marr and S. Helme, 1991

Squaring the Triangle



Mount onto card and cut out the four shapes above.

Jumbled Jobs

What are the names of the only electrician, lawyer and doctor in Werribunyip?

- The electrician, doctor and lawyer are called Melissa, Laura and Gino, but not necessarily in that order.
- Gino is the lawyer's neighbour.
- The electrician is Gino's daughter.
- Laura and the lawyer have the same birthday but one is 25 years older than the other.
- The doctor treated Laura for a burned hand.
- Melissa and the electrician often swim together.
- Melissa and Gino were in the same class at high school.

Use the grid and cards below to help you solve the problem.

Jumbled Jobs

	Melissa	Laura	Gino
Electrician			
Lawyer			
Doctor			

✓	✗
✓	✗
✓	✗
✓	✗
✓	✗
✓	✗
✓	✗
✓	✗
✓	✗

Colour Logic

Fill in the correct colours from the clues given below:

- The blue is on the right of the green.
- The brown is next to the red.
- The green is in front of the yellow.
- The blue is on the right of the black.
- The brown is next to the yellow.
- The red is behind the blue.

Use the grid and the hands-on materials to help you solve the problem.

Handout 4.4B p.11

Breaking The Maths Barrier, B. Marr and S. Helme, 1991

Colour Logic

(back)

(front)

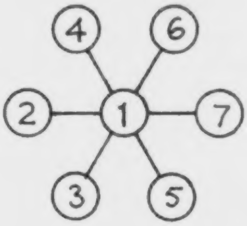
Use the grid to solve the problem above.

Answers to Problem Solving Stations

Cats and Budgies

Gina and Tom have 3 cats and 7 budgies.

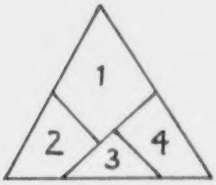
Round in Circles

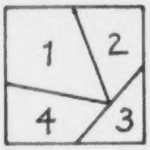


Working out the Roster

	Monday	Tuesday	Wednesday	Thursday	Friday
a.m	Frank	Joan	Cathy	Bridget	Cathy
p.m	Pat	Pat	Joan	Bridget	Frank

Squaring the Triangle





Jumbled Jobs

Electrician:	Laura
Doctor:	Gino
Lawyer:	Melissa

Colour Logic

yellow	brown	red
green	black	blue

218

Activity 4.5 **Experiencing Fractions**

Rationale

These activities serve two major purposes. Firstly and most importantly, they allow participants to see how effective hands-on materials can be to an understanding of important mathematical concepts. We believe that teachers and tutors will be enthusiastic about using such materials with students only after they have first experienced them for themselves. Secondly, the activities chosen are designed to improve participants' own understanding of basic fraction concepts. Several times in this activity participants use the technique of role playing: taking student and teacher roles to put themselves in a situation where they must clarify their own ideas by verbalising them to another person.

Aims

To allow participants to experience one example of hands on materials and its use in exploring:

- the understanding of the concept of fractions of a whole entity e.g. 2/3 of a cake;
- visualisation of commonly occurring fractions;
- language associated with fractions;
- the concept of equivalent fractions;
- addition of simple fractions (including some with different denominators).

Materials and Equipment

- 7 different colours of A4 card (See Preparation)

Time 45 minutes to 1 hour

Handouts

- 4.5A *Chart for Experiencing Fractions*
- 4.5B *Experiencing Fractions: Participant worksheet*
- 4.5C *Fraction Kit Outlines*

Preparation

You will need 7 different coloured pieces of card. Photocopy the fraction kit outlines onto coloured card; one colour for each fraction type. Preferred colours consistent with other kits are:

Fraction	Colour	Fraction	Colour
Whole	White	Fifth	Yellow
Half	Red	Eighth	Brown
Third	Light Green	Twelfth	Purple
Quarter	Pink		

For the most efficient workshop you should prepare one kit for each pair of participants. Cut these out and place each kit in a separate envelope.

Conducting the Activity

These activities are best done with participants in pairs so that they have plenty of chances to use the pieces, use fraction language and discuss their reasoning. (If you haven't enough kits for pairs, keep groups as small as possible.)

Give out one fraction kit to each pair. Ask them just to examine it and play with the pieces for a short time. You may want to encourage them to make up circles of one colour. You may also wish to begin the activity as a group to get them started. This will depend on the group.

Now hand out one copy of Handout 4.5A *Chart for Experiencing Fractions* and Handout 4.5B *Experiencing Fractions, Participant Worksheet* to each pair of participants.

Not all participants will come with the same background knowledge or understanding. Working in pairs means they can progress at a rate suitable to themselves and explore the aspects of the activities which are most interesting to them. Do not expect all pairs to finish the sheet in the allotted time. If they wish to, participants can continue their discoveries later.

Depending on the size of the group there are alternate ways of proceeding with this activity.

For a small group you may wish to conduct most of the discussion with the pairs as you move around the room and sum up with just a few closing remarks at the end (see Concluding Discussion). **For a larger or more homogeneous group** you might interrupt their deliberations at regular intervals for a group discussion.

In any case you should circulate to observe and be available for questions, and ensure that most of the aspects presented below are covered.

Discussion of Activities

Activity 1: Naming the Pieces and Explaining the Symbols

For example, one red piece is represented by 1/2 because it takes exactly two red pieces to make the whole circle and this is one of them.

The words: Note that the first few fraction names do not have a logical system to them as do the later names such as one eighth, one ninth etc. The words half, third, and quarter, can be confusing to students, particularly those of non-English speaking backgrounds. Time spent with students discussing the words, the pronunciations, and comparisons with other language systems can be very valuable.

Numerator and Denominator: The bottom number, denominator, tells you the name of the fraction. In the case of our model it identifies the **colour** of the pieces. The top number, numerator, tells you how many, or the number of, identical pieces you have. (You may wish to discuss the relationship of the word denominator and uses of the word denomination.)

Whether participants choose to use and discuss these terms with their students is up to them, but this discussion of the words might make them easier to understand.

Activity 2: Ordering

You are looking for a general rule for single pieces which states something like:
the fraction pieces get smaller as the number on the bottom gets larger.

The pairs chosen in the exercise cover each of these categories, highlighting possible student mistakes.

1.	Same numerator but different sizes of fractions	3/4, 3/8
2.	Same denominator but different sizes of fractions	1/3, 2/3
3.	Bigger denominator but smaller fraction	1/2, 5/12
4.	Bigger denominator but larger fraction	1/4, 3/8
5.	All numbers different but same size fractions	2/8, 3/12

Activity 3: Equivalence

Note we are not necessarily, in basic numeracy, looking for the conventional rules which lead to cancelling fractions. Students usually come up with rules such as that for 1/2 = 2/4 , 3/6, 4/8, or 6/12: *the number on the top is always half the number on the bottom.*

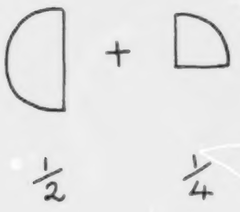
This also works for 1/3 = 2/6, or 4/12, where the **numerator is a third of the denominator.**

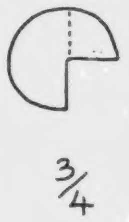
For this stage of fraction development rules of this type make sense and are useful for consolidating the concept of a half, a third, etc. The main thing we are trying to achieve here is the idea that there are many ways of expressing the same fraction quantity.

Teachers who wish to carry this concept further with their students can be referred to a detailed development in *Mathematics: A New Beginning*.

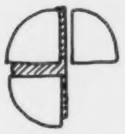
Activity 4: Addition of Simple Fractions

When adding 1/2 and 1/4 the answer is obviously 3/4 because of the familiar shape which can be seen:





People are used to this shape because of the 3/4 hour configuration on the clock face. However to demonstrate clearly there are three of the one quarter pieces it is necessary to cover the one half piece with the two quarter pieces:



in other words the half becomes two quarters so 1/4 + 1/2 =

1/4 + 2/4 = 3/4

Concluding Discussion

At the end of activities bring the group together to discuss the activity. Ask for comments or feedback and explore or clarify if necessary the issues raised by your observations and discussion with participants as they undertook the activity.

Questions you could ask:

- *Were the hands-on activities useful? How?*
- *How important is language in learning about fractions?*
- *What have you learned from this activity?*

Further references

Marr, B. and Helme, S.(eds. 1987) *Mathematics: A New Beginning*. State Training Board Vic.

Goddard, R. Marr, B. and Martin, J. (1991) *Strength in Numbers*. Holmesglen College of TAFE Vic.

Both of the above publications are available from the Teaching Maths to Women Project, Northern Metropolitan College of TAFE, St. Georges Rd Preston Vic. 3072

Handout 4.5A

Chart for Experiencing Fractions

Number of Pieces	Colour of Piece	Fraction Symbol	Written Name
One		$\frac{1}{2}$	Half
One			
One			
One			
One			
One			
One			
Two			
Three			

Experiencing Fractions: Participant Worksheet

To be done in pairs after you have had a few minutes to examine the fraction kit.

Activity 1: Names and Symbols

Person 1 Pick up a single fraction piece. Make a full circle from all the pieces of the same colour. Then decide what the piece will be called and write the **colour**, the **name**, and the **symbol** which represents it, in the chart provided.

As you do this imagine that your partner is a student with little concept of fractions and explain in detail the meaning expressed by the symbol.

Now, swap roles.

Person 2 Pick up another piece of card and repeat.

Continue, swapping roles from teacher to student as you go, until the chart is filled in for each colour of the kit.

Together Look for the names in the last column and briefly discuss problems which they might present to your students. Is there a logic or pattern to these words?

Again sharing the tutor/student roles:

Person 1 Pick up **two of the 'one third'** pieces and place them together.

What fraction do you have? How is it said or written?

Fill in the chart.

Person 2 Model **three eighths** with the pieces, and fill in the chart.

Person 1 Model and name **5/12**, filling it in on the chart.

Person 2 Pick up **four of the one fifth pieces** and fill in the chart for them also.

Together Look at all the fraction symbols you have written and discuss:

What does the bottom number tell you?

What does the top number tell you?

Can you make any sense out of the terms numerator (for the top number) and denominator (for the bottom number)?

Experiencing Fractions: Participant Worksheet

Activity 2: Ordering

Together Take one piece of each colour and arrange them from smallest to largest.

Write the associated fraction symbols in the same order.

Express this as a fraction law or rule written in your own words.

For the following pairs of fractions:

First **circle** which you think will be bigger.

Then **check** by modelling with the fraction pieces.

As you do each pair discuss possible misconceptions which may be held by students and lead to the wrong choices.

1. 1/8, 1/12 2. 3/4, 3/8 3. 1/3, 2/3
4. 1/2, 5/12 5. 1/4, 3/8 6. 2/8, 3/12

Activity 3: Equivalence

Together Make as many 1/2 shapes as you can using pieces of a single colour.

From these **write down** all the fractions you have made which are **equivalent to** (the same size as) one half.

For the pairs below:

First **guess** whether they are equivalent, **discussing** your reasons.

Then **check** with the pieces.

1. 4/8, 1/2 2. 4/12, 1/3 3. 1/4, 2/8
4. 1/2, 3/5 5. 3/4, 6/8 6. 1/3, 3/12

Activity 4: Addition of Simple Fractions

Person 1 Use the pieces to model the calculation:

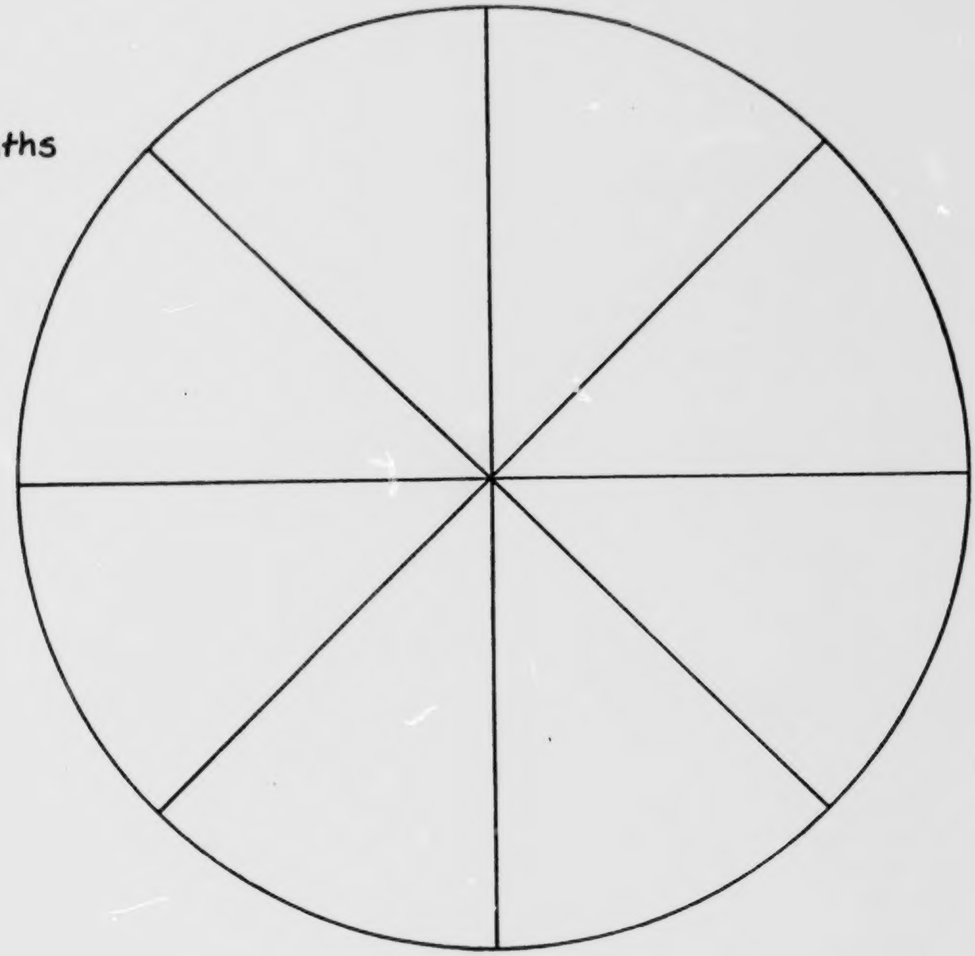
1/4 + 1/2 =

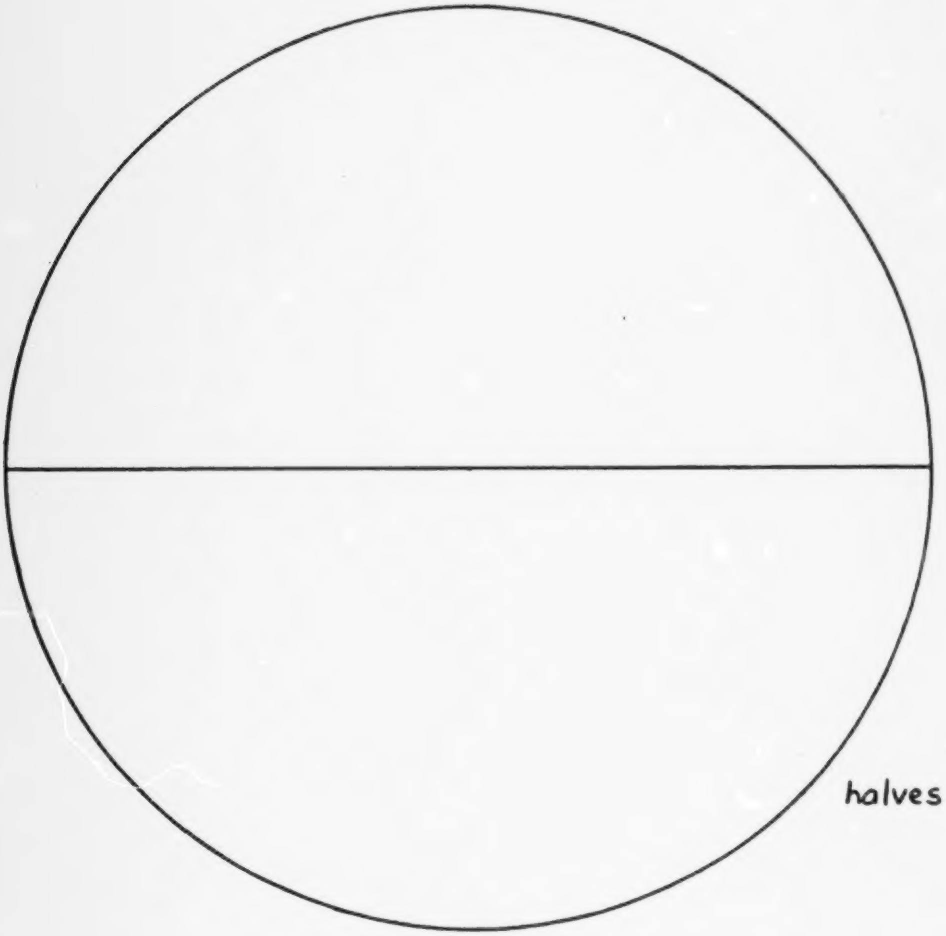
Write down the answer (i.e. the shape you see). Clearly explain and demonstrate this answer to your partner.

Reverse roles

Person 2 Repeat above for 1/4 +1/8 =

eighths



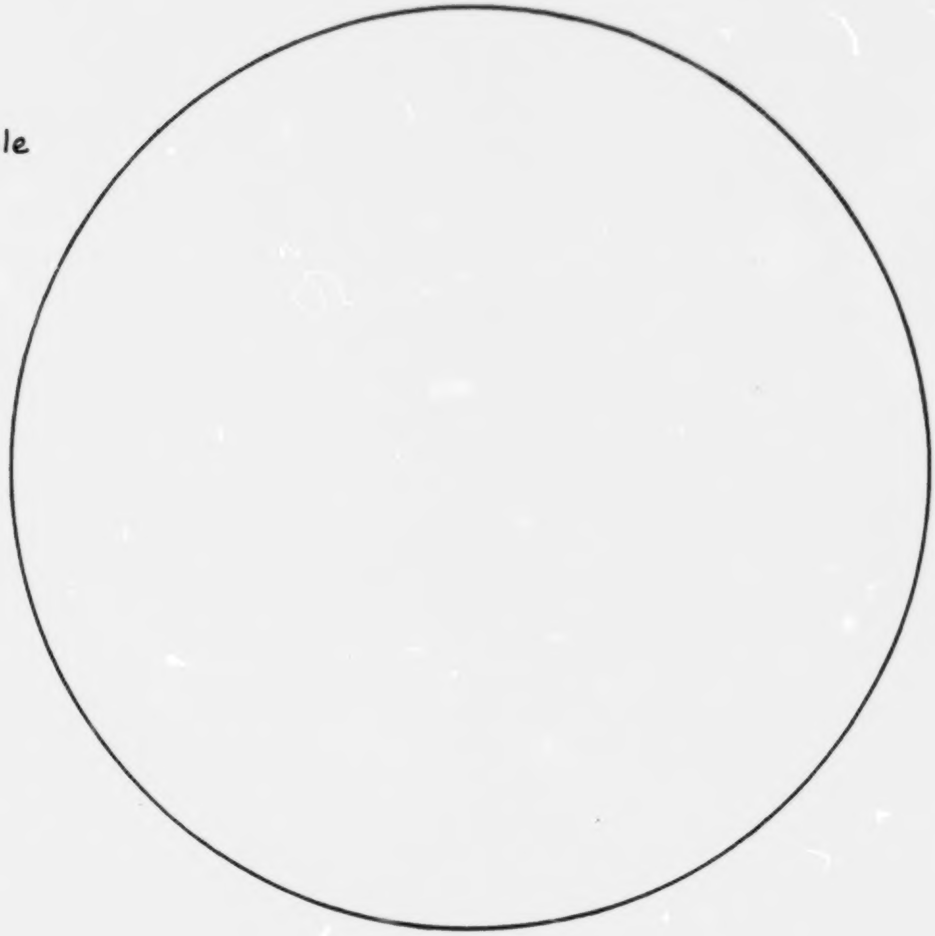


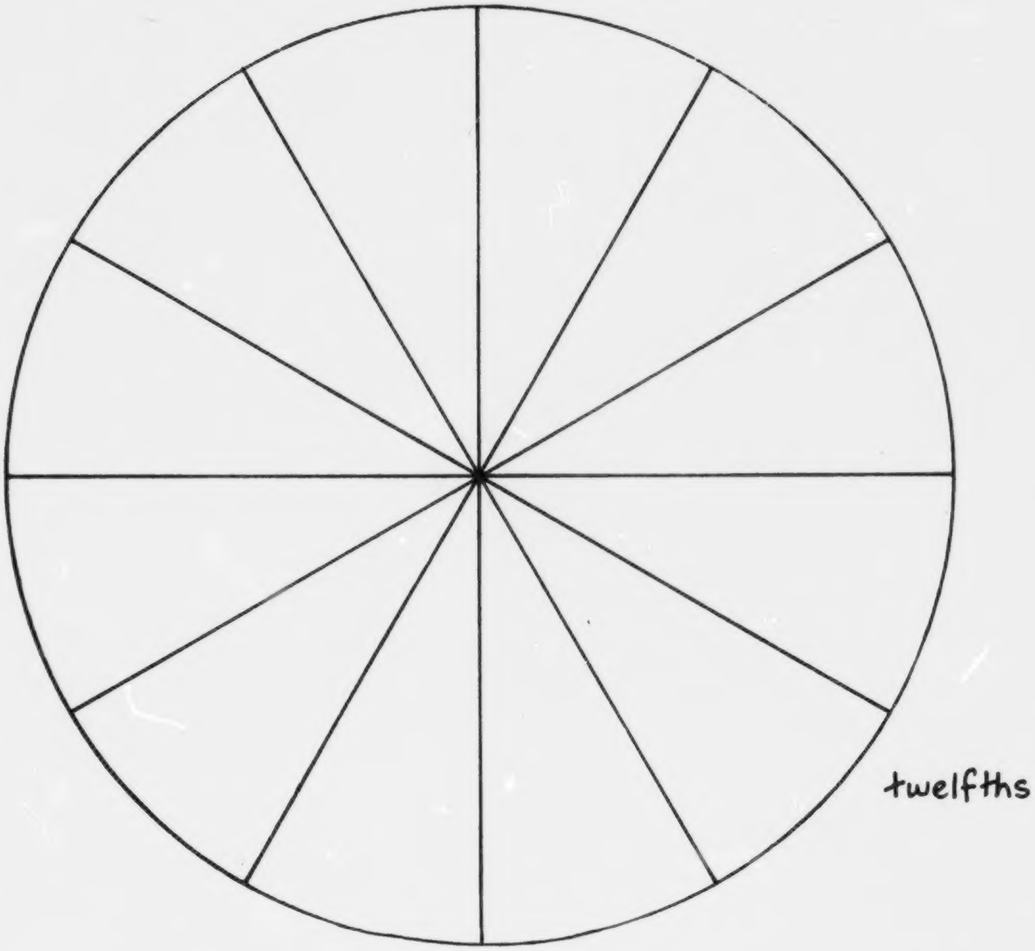
228

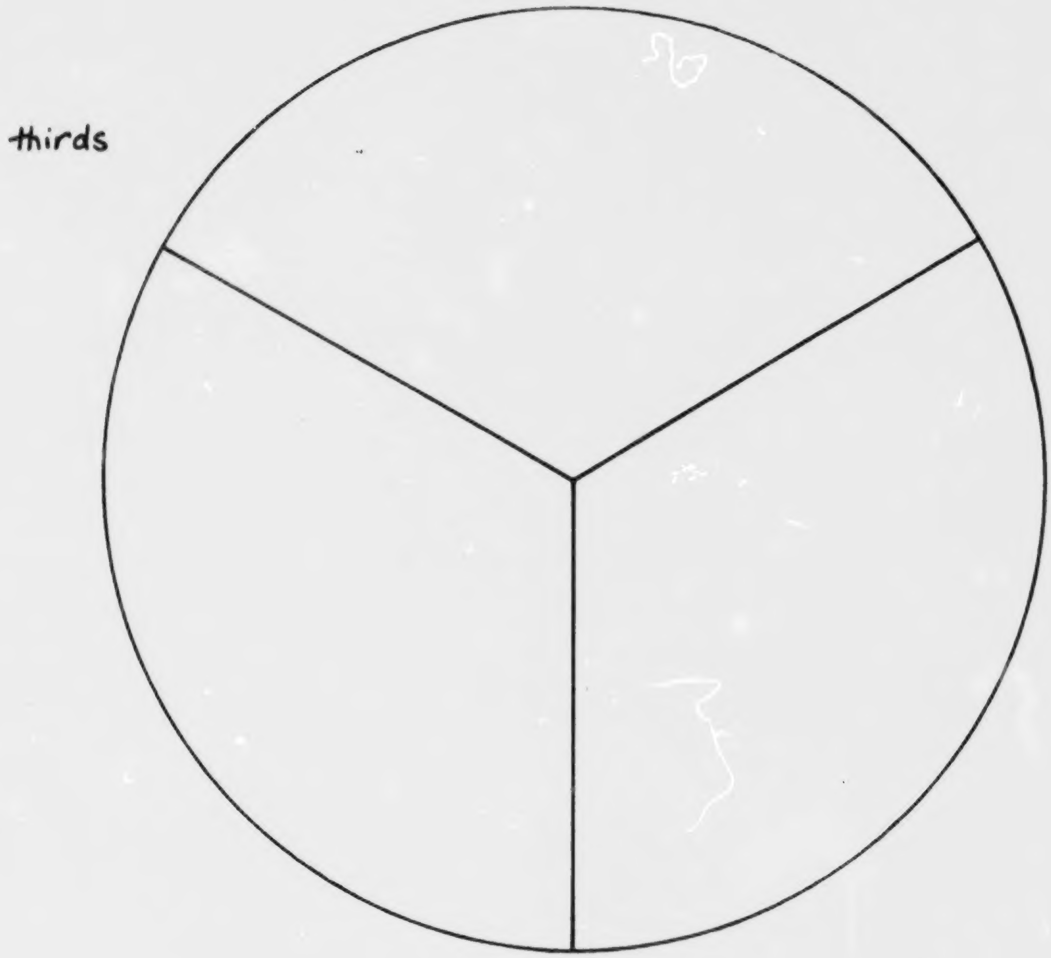
Fraction Kit Outlines

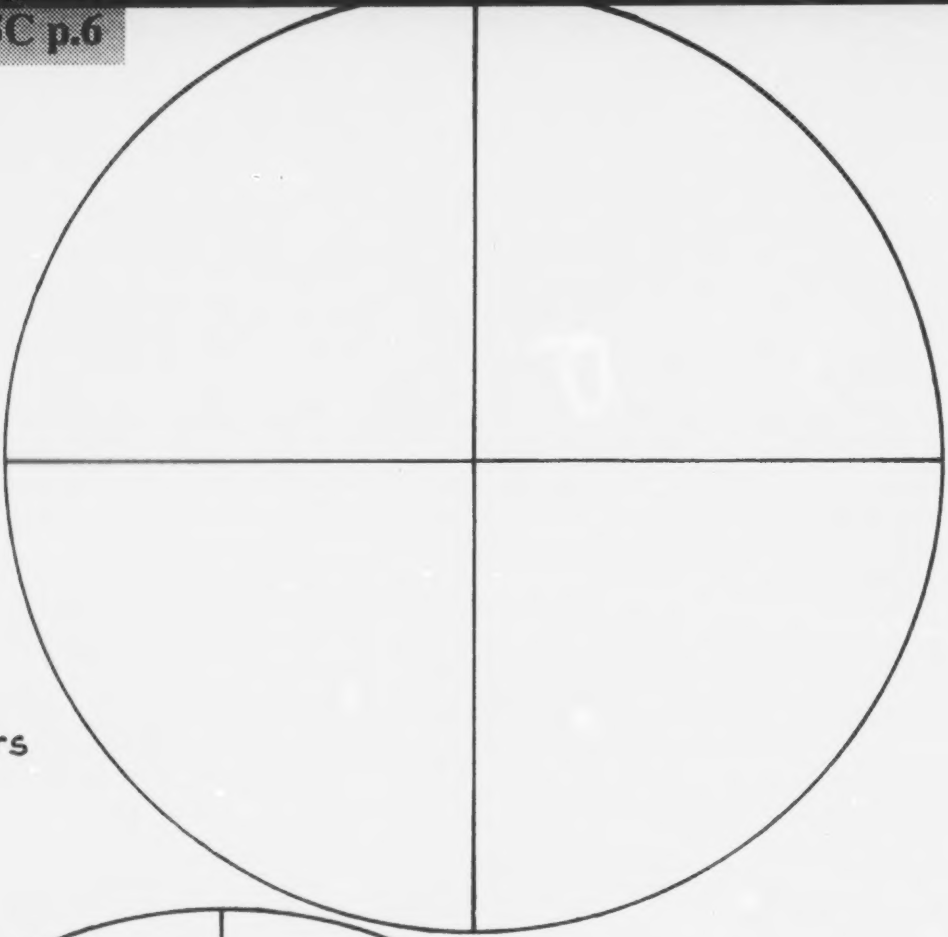
Make 2 copies of each outline.
Mount these together on one A4 page.
Photocopy onto coloured card. (Or simply trace and press through to coloured card.)

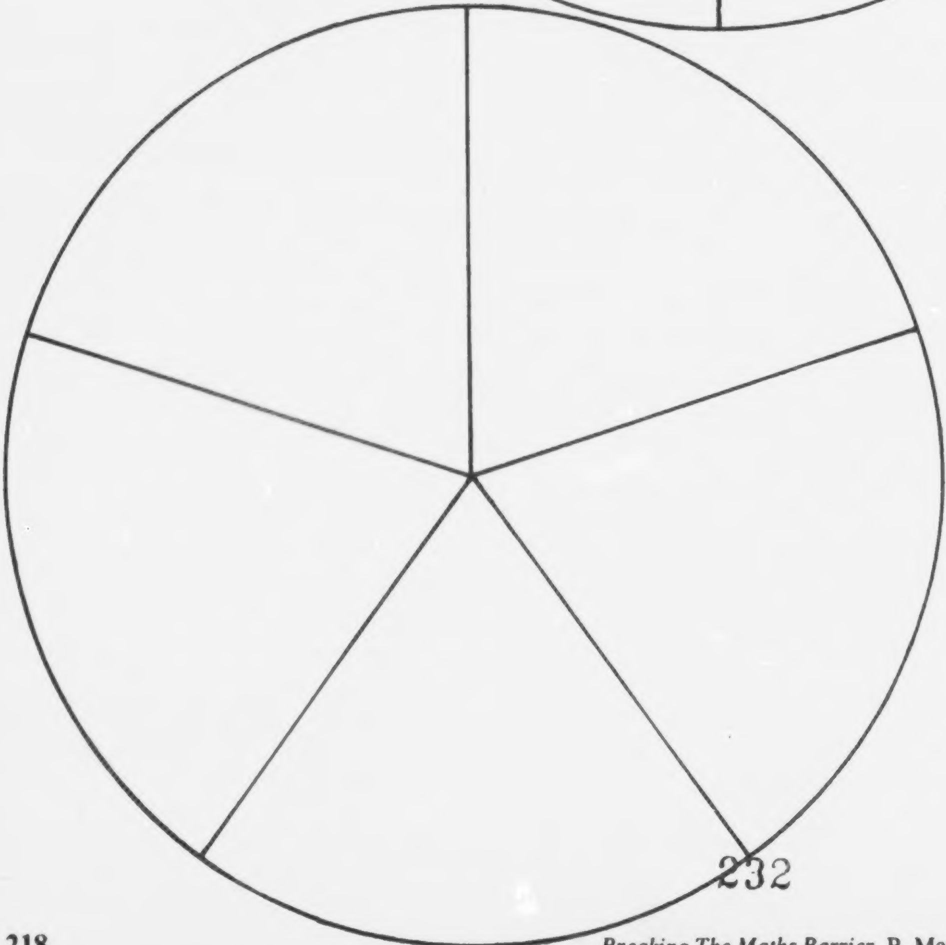
one whole




twelfths


thirds





232

Interaction and Cooperation

What is Cooperative Learning?

The advent of cooperative learning is probably the most innovative change in mathematics teaching and learning in the past decade. The plethora of papers and books now written on the subject of cooperative learning describe its advantages and define different types of cooperative learning, such as cooperative logic, which have become buzz words in educational practice and research.

For our purposes we will describe cooperative learning activities as those performed by a small group of two or more people working together to reach a common product. This could be a solution of a problem, the collection of information and production of a graph, an estimation and measurement activity, or exploratory work with hands-on materials such as fraction circles or place value equipment in order to understand a concept.

Why Cooperative Learning?

The benefits gained through the use of cooperative learning techniques fall into two categories: increase of mathematical learning; and development of students' personal skills. We discuss both of these aspects in the following pages.

Advantages for Learning Mathematics

- **Learning through communication**

  Cooperative learning is used because it offers opportunities for students to **learn new concepts and skills through communication** with each other. In groups small enough to break down the barriers of shyness and lack of confidence, students learn through talking and listening to each other.[1] They can clarify their own ideas by expressing them out loud to other students, gain by listening to the alternative approaches of fellow students, and consequently developing each others' ideas further. Students can also assist each other to articulate their reasoning by asking clarifying questions.

- **Reduction of anxiety**

  Being part of a group working towards a common product can remove the feelings of isolation and panic so often experienced by adults in past maths learning situations. From experience we know that students returning to study may well remember maths classes in which they were required to sit quietly and to passively listen, and where to look at another person's work was cheating. One sat alone with paper and pencil struggling to understand the problem, or worse, to get the right answer without really understanding the concepts and processes being used.

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[1] Though we refer here to talking and listening we do not mean that non speech and hearing impaired students will not benefit from such activities. Communication can also be by computer, sign language, communication boards, or whatever means people have developed to express their thoughts to one another.

In the introduction to his book *Cooperative Learning in Mathematics* (1990) Neil Davidson refers to that experience:

> *This process can often be lonely and frustrating. Perhaps it is not surprising that many adults are afraid of mathematics*(p.3).

Group work removes this isolation and replaces competition with cooperation. It engages students actively in the learning process, completely changes the atmosphere of maths learning recalled from their 'bad old days', and creates a more relaxed and informal classroom atmosphere.

- **Reduction of student anonymity**

Small groups also help to **break down the feelings of anonymity** felt by members of larger classes. When the teacher interacts with a small group she/he is able to make eye contact with each individual and concentrate on each of their responses. This way each group member has a feeling of having received individual attention.

- **Development of mathematical language**

Group work is particularly valuable for the **development of the language associated with mathematical concepts.**
For example in cooperative logic activities a great range of basic vocabulary and mathematics concepts can be introduced or reinforced. These logical problem solving activities are structured so that students need to read the words on the clue cards out loud and understand their meanings in order to reach a solution. Thus they will explore and explain to each other new concepts and language out of a genuine and immediate desire to know, and in a specific context. This is much more effective than a blackboard vocabulary lesson where the teacher does most of the talking and explaining.

Such activities are, of course, extremely useful in the teaching of adults from non English speaking backgrounds. Research by educational researcher, Richard Skemp (1987) supports the observations of teachers of both adults and young students that students learn more effectively when they have attempted to convey their reasoning in words to another person (p. 61).

- **Increased student verbalisation and participation**

Once we accept that verbalisation assists learning, it is easy to appreciate the advantages of having a **number of small groups** in the room as opposed to one large group. The **amount of verbalisation taking place is increased** and the shy, more reserved class members are less afraid to contribute.

- **Increased level of mathematics**

Another advantage of groups is that students in groups can often **handle more challenging tasks** than they would be able to do on their own. Individuals working alone may give up when a problem looks a bit complex. However, with group support and mutual encouragement they will persist with the task.

- **Increased feedback**

  Having students explaining to each other also **allows the teacher to move around** and listen to the verbalisation of students' thought processes. This is a most effective way of picking up misconceptions which the teacher can then address, either on the spot, or later with the entire class, if such misunderstandings appear to be common.

Other Advantages of Cooperative Learning

There are other advantages to be found in cooperative group work besides the actual mathematical learning and language development taking place. Group work can also help to:

- **develop social skills** and effective interactive behaviour in the students, because they are learning to work with other people. These include skills such as attentive listening, explaining ideas to others, encouraging others to participate, and group decision making.
- **foster interaction between class members** by changing groups so that all class members work with people they would not normally mix with. (Both these points can be especially useful when integrating students with disabilities into the class).
- **encourage the idea of completing a task**: support and energy is available where one person alone might have given up.
- **develop the confidence of shy students** who may be more inclined to speak in a smaller group.
- **make students take more responsibility for their learning** and that of other group members and encourage them to trust their own judgment. Thus they become less dependent on the teacher.

How to Make Cooperative Learning Work

The advantages of cooperative learning outlined above cannot always be guaranteed without some conscious work on the part of the teacher. Although some students will naturally interact and learn from one another whatever the class format, **well structured group** work assists **all** students to reap the benefits of group activities.

Introducing Group Work

Attention needs to be paid to how you introduce group work to adults for the first time, the type and presentation of tasks given to the group, and the interpersonal dynamics operating. As Neil Davidson (1990) says:

> *Cooperative learning involves more than just putting students together in small groups and giving them a task. It also involves very careful thought and attention to various aspects of the group process* (p.1).

Some Problems with Group Work

The main problems which seem to occur in groups of adults with improperly structured tasks are:

- domination by louder, more confident group members who may monopolise resources and equipment, or do all the talking, often at the expense of shyer people who become even less willing to express their ideas.
- some students becoming lost while faster students rush ahead. Thus they may disengage from the activity and learn little from it.
- quicker thinking students feeling held back by slower students instead of seeing the opportunity to explain their thoughts to others as assisting their own learning process.
- some students 'doing their own thing' or working totally individually at a group table, so that they are not using language or interacting with other group members, thus defeating the purpose of cooperative work.

Usually only one of these problems will occur at any one time and we offer suggestions about how to counteract such situations in the following section.

Strategies for Group Work

Begin with Pairs

It is a good idea to begin the process of interaction using pair activities.

Some suggestions for initial pair work are:

- Turn and talk to the person next to you for two minutes about a given topic such as:
  - something you remember about school maths classes.
  - how you would describe a given shape or object.
  - how you would tackle a particular sum or problem.

  (This is a method recommended by many educators to accustom students to sharing their ideas and listening to those of others.)
- Solve a problem or do a calculation together making sure the solution is agreed to by both.
- One person describes a geometric figure or pattern whilst the other attempts to draw it from the description.
- Pair interviews in which one person interviews the other for about five minutes and then reports back. The topic could be a single question or a selection of questions. It could explore past experiences of maths, feelings about maths, or explore specific mathematical topics. Some examples are:
  - *describe a situation when you use maths at home.*
  - *what do you remember about learning long division?*
  - *tell me what the word 'fractions' make you think of.*

Such activities assist in developing students' listening skills and provide practice at summarising and reporting back for later group work.

Moving Students into Groups

Since student numbers in numeracy groups are usually small you won't want to make a big issue of selecting or allocating students to groups. However some strategies can be useful to think about.

- Friendship groups or pairs often occur naturally for adults in basic education classes. These should be encouraged and maintained initially for working groups. If, after a time it appears that one student is relying too heavily on a friend or group, and becoming too dependent on them, or some students are not integrating into the class, then you may wish to try other strategies.
- When groups of four are being introduced, a useful mechanism is to get a pair who have already been working together to select another pair to form a four.
- Random groupings can be organised by students counting off 1, 2, 3, 4, 5, 6 so that odd numbers go together or multiples of 3 (3, 6, 9, 12) go together. This is especially useful if you wish to separate people who usually sit together.
- Shuffling cards and matching with a pair or group of four can be useful. For example all aces get together or red fours get together. Of course you will need to choose which cards you include so that the matching is possible.
- If you want groupings to be closer in ability you can group students who are working on similar material.
- Sometimes you need to be aware of personalities and give the quieter people a go by putting them together while the more dominating people work together. This will however, not necessarily be seen by the students to be as fair, or random as other methods are. It depends on the group whether or not that aspect matters to them.

Structuring the Tasks

By structuring group tasks in a variety of ways you can facilitate effective co-operation and interaction. Some of the strategies for doing this are:

- the **jigsaw model,** in which each member of the group is given their own information to contribute to the group task. This method ensures that each student participates to some degree in the activity. Cooperative Logic is a classic example of this method.
- use of **hands-on materials** in the centre of the group makes the active solution more visible and accessible to all group members than it would be if paper and pencil were used. When using only paper and pen solutions there is always the tendency for someone in the group to monopolise the action.

- give only **limited resources** to the group, so they **must** share, e.g. only one chart to fill in with a report at the end will counteract the tendency for group members to work alone when a discussion or brainstorming is more appropriate.
- **seating** in such a way as all students in the group can see and hear each other easily, reach the materials, and read any documents used. Smaller tables, or seating in circles, lend themselves better to group work than long rectangular tables.
- **keep it short and simple.** Certainly in the early days you should make sure that the tasks can be completed easily in one session.

The main advantage of the structuring approach is that it ensures simple cooperative skills will be used and practised without taking extra valuable time to concentrate on them. Thus the students will reap many of the learning benefits of group work without a lot of time spent on discussion.

This makes the structured approach ideal for adult groups with whom you have limited contact, such as short courses or those with very few hours each week.

The disadvantage, however, is that students will not necessarily develop group skills in such a way as to be transferable in other situations: those situations in which the structure is not specified by somebody else.

The alternative is to actually spend time developing group skills with your students.

Motivating Students to Learn Group Skills

In order to spend time developing group skills in your students you will first probably need to convince them that it is a worthwhile enterprise.

You could begin with the words of Australian educator Joan Dalton from her book, *Adventures in Thinking:*

> *What could be a more basic skill than cooperation? It is not only a powerful way to learn, but it is basic throughout life for developing positive, working relations, maintaining stable families, and developing effective community membership* (p.10).

Remind students that most adults have to work in groups at some stages in their lives. It may be as a parent on school committees, as a member of a tennis or football club, perhaps participating in self help groups to organise political activities, or on the job in a work team or as part of a union. Effective participation in such groups requires many personal skills such as:

- listening to others in a sensitive way;
- putting forward your ideas in an effective manner;
- encouraging others to participate;
- group decision making (e.g. reaching agreement on a solution to a problem);

- resolving conflict;
- giving and receiving constructive feedback.

As well as the more practical skills of :

- taking notes;
- summarising discussions;
- reporting your findings to others.

Aside from the actual personal interactive skills which group work develops there is evidence which suggests that group work is extremely effective in **helping students learn.**

Long term advocates and practitioners of cooperative learning, Ted and Nancy Graves have a lot to say on this subject in their book, *A Part To Play* (1990)

> *Cooperative learning research devoted to providing practical classroom applications began in the early 1970's. The scope of research is extensive. It has taken place in different countries, inner city, suburban and rural schools from grades two to twelve and in subject areas ranging from mathematics to language arts, to social education and reading. In cooperative classrooms students consistently score higher in academic achievement, have improved intergroup relations, increased self esteem and increased liking for school, class and subject.*
>
> *They also develop the positive social values and behaviours that other types of instruction do not affect* (p.23).

And Joan Dalton:

> *From the work of Johnson and Johnson (1975) and others, it is apparent that learning cooperatively in small groups makes a dramatic difference to a range of learning outcomes. It promotes:*
>
> - *higher achievement and creative productivity;*
> - *divergent thinking and effective problem solving;*
> - *thinking skills at higher cognitive levels* (p. 9).

There is one further motivator worth mentioning for bridging students, or any students thinking of proceeding to further study. It is now common practice in tertiary courses to focus on group work, group discussions, and to assess students via group assignments. Learning effective group skills early can only be an advantage to your students.

Building Group Skills

To work effectively in cooperative groups, students need to develop appropriate interpersonal communication skills. Increasing their awareness of their own behaviour and how it affects the group is a good starting point.

One method of introducing discussion of the kinds of dynamics and roles which emerge in groups is to encourage students to analyse the behaviours which are characteristics of group work by observing their own behaviour and to monitor changes in themselves and their groups over a period of time performing group problem solving tasks. This may be all that many teachers will have time to do, since, unlike the school situation, our time with many students is very brief.

Others may, however, wish to do more to facilitate positive changes especially if the group is participating in a long term or full time program geared towards seeking employment or integration of disadvantaged students into mainstream courses.

A few suggestions are presented here. They are however mere **snippets** from a number of books devoted to the subject which are worth delving into.

Assigning Roles

The practice of students acting out a series of prescribed roles has become a widely and effectively used technique in many learning situations, not only with children but with adult students as well.

Joan Dalton, in her work with primary students, advocates the gradual introduction of a range of skills. She concentrates on one or two skills at a time by assigning particular roles to students in a structured program of activities.

One example, an initial activity, has students **in pairs** taking alternatively the roles of **talker** and **listener** whilst discussing a prescribed subject The listener does not interrupt at all but when the short time interval is over has to report back to the talker what she/he heard. This highlights the need to really listen so as not to have gaps in the reporting back. The skill of reporting back is further developed by asking listeners to report to a whole class.

Dalton then builds gradually on this activity by introducing the skills of **asking clarifying questions,** e.g. ask two questions about her story. She assists children in this process by introducing the 5 W's and H: Who, What, Where, When, Why, How (p.12 -13). Later, she assigns to pairs joint tasks on which a single outcome must be agreed. This introduces the ideas of **single outcomes, joint goals** and begins the process of **reaching consensus.** Over a period of time she moves the pairs into groups of four and expands the repertoire of roles students practise to include:

Recorder	who records the ideas and decisions of the group.
Reporter (or spokesperson)	who reports back to the larger group.
Encourager (or facilitator)	who ensures that everyone in the group participates and contributes ideas.
Time watcher (or director)	who keeps the group on task.

Brainstorming with students about what such roles might entail enables them to become aware of and practise new behaviours and modes of speech until they become natural .

As well as developing skills which are not part of their usual repertoire students may develop greater awareness of the role they tend to play naturally in group situations. For example, a female student for whom the roles of encourager and recorder are familiar may not be so comfortable with the role of reporter, but discussing and practising this new role may assist her to develop the skills and confidence it entails.

When roles are used by groups they should of course be rotated to give everyone a chance to practise new skills. It is also important that each activity be followed by a short time for reflection where the members of the group can discuss how well each fulfilled their assigned roles. This might be built into the group activity in a structured way.

One suggestion might be asking the group to decide what they did well, what they could have done better and one thing they would like to concentrate more on next time.

It can also be a good idea if members compliment each other on one way that they assisted with the group task each time.

Many of the aspects discussed in this introduction are covered in a video *Cooperative Learning: From Me to We.* The video shows many of the ideas discussed put into practice in school situations and could be used as a discussion starter or as a focus for review after cooperative learning has been explored.

References

Neil Davidson (Ed.1990) *Cooperative Learning in Mathematics - A Handbook for Teachers*. Addison Wesley.

Nancy and Ted Graves, (Australian Editor Kathy Lacey 1990) *A Part to Play: Tips Techniques and Tools for Learning Cooperatively*. Latitude Publications.

Joan Dalton (1985) *Adventures in Thinking- Creative Thinking and Cooperative Talk in Small Groups*. Nelson.

Richard Skemp (1987) *The Psychology of Learning Mathematics*. Penguin.

Video *Cooperative Learning: From Me to We*. (Available for sale from the Education Shop, $11.50 plus $4 postage, or for loan from the Film and Video Loans Collection, both at 117 Bouverie St. Carlton, or by mail, P.O Box 645, Carlton South Victoria. 3053 Tel 03 342 3939.)

The Activities

Activity 4.6 ***Cooperative Logic*** is a session which allows participants to experience and discuss a wide range of cooperative logic problems: i.e carefully structured problem solving activities designed for group work. The problems selected for inclusion in this session specifically focus on spatial concepts. They are taken from a number of different resource books, so participants can be made aware of major sources of alternative problems for their students. Also, should the presenter prefer, they can select other problems to present in the session. The session focuses, not only on the problem structure but on the strategies, skills, and knowledge used to solve them, and how to assist student development in these areas.

Activity 4.7 ***Exploring Groupwork with Problem Solving*** allows participants to examine two aspects of mathematics/numeracy teaching concurrently. It presents a range of problem solving tasks in order to facilitate discussion of problem solving strategies and how to teach them. The activity also sets up group work situations so that participants can think about, and discuss methods by which students interactions can be improved.

Activity 4.6 Cooperative Logic

Rationale

Cooperative logics are an excellent form of structured group work. They encourage cooperative learning, development of problem solving strategies, and development and reinforcement of mathematical concepts and the language associated with them.

There is a wide range of cooperative logic problems available. The selection we present here is particularly useful for the development of everyday spatial concepts.

As with many of the techniques outlined in this publication, it is vital that teachers see these activities in action to give them the confidence and assurance to use them with their own students.

Aims

- To allow participants to experience a range of cooperative logic problem solving activities.
- To develop participants' problem solving strategies, and methods for developing such strategies with their students.
- To increase awareness of the spatial concepts and related language which should be addressed by teachers of literacy and numeracy.
- To provide a focus for the discussion of group work; including its benefits and methods of structuring it for maximum participation.

Materials and Equipment

- 1 copy of each problem made up for each small group of 4-6 participants (see preparation)
- 1 set of coloured cubes for each small group: 2 red, 2 yellow, 2 green, 2 blue, 2 orange. These can be centicubes or wooden cubes. Even small MAB cubes coloured with textas will do.

Time 1 hour

Handouts

1 copy of each of the activities used for each participant.

- 4.6A *City Block*
- 4.6B *The Flats*
- 4.6C *School Fete*
- 4.6D-F *Build It 1, 2 and 3*
- 4.6G *Publications Containing Cooperative Logics*

Preparation

Make one copy of each problem for each group of 4-6 participants. Glue each sheet onto card, cut out the clue cards and moveable pieces. Store each problem in an envelope and label it.

Conducting the Activity

Divide participants into groups of four. If necessary groups can go up to six in number but this is not desirable for maximum involvement of all participants.

Preliminary Discussion

1. Explain to participants that these cooperative logic activities are for a range of students of varying language ability. We would like them to experience the activities for themselves in order to fully appreciate the problem solving skills, language and concept learning embedded in the problems.

2. Mention that cooperative logics are available at more sophisticated levels of mathematics (see references below) but in the selection presented here we are catering for the numeracy/literacy and basic mathematics levels. *School Fete* and *Build It* require a more complex reading level and introduce slightly more sophisticated mathematical vocabulary and concepts such as adjacent, edge, and face.

3. Ask participants, whilst doing the problems, to reflect upon the possibilities of using these or adapted versions with their own students - how the activity could act as a springboard, or stimulus for a sequence of learning tasks or lessons in a literacy, numeracy or mathematics class. If time permits at the end of the problem they should discuss their ideas.

Explaining the Rules

- Each student should be given at least one clue card. In some problems there are starred cards which can be omitted in the first round and used for checking i.e. *Build It, School Fete* and any EQUALS Publication problems. For others (no starred cards) all the cards are needed and should be distributed, so some group members will receive two cards.

- No one can give their clue cards to another person or lay them down for others to read. They can however, read their card **aloud** as many times as they, or the group needs, in order to solve the problem.

- Give out one problem at a time to each group. For a group of non mathematics trained teachers start with the simple *City Block* and work towards the more difficult ones, such as *Build It*. It may be a good idea to use the opposite approach if working with a group of maths trained people, i.e. challenge their problem solving skills early with absorbing mathematical problems and then progress to problems more suitable for beginner students. Mathematical problems from some of the publications mentioned in Handout 4.6G would be useful, though *Build It* and *School Fete* can be quite challenging.

As the groups work on their problems you should circulate and perform a number of roles:

- initially, clarifying rules and procedure;
- giving out new problems;
- discussing the activities as the groups complete them with questions such as:
  - *Did you like it?*
  - *What were its good features?*
  - *Would it be appropriate for your students?*
  - *How could it be modified ?*
  - *What other activities could flow from it?*
  - *How would you have solved it without manipulable pieces?*
  - *Why do you think we provided cut out numbers rather than pencil and paper?*

Round Up

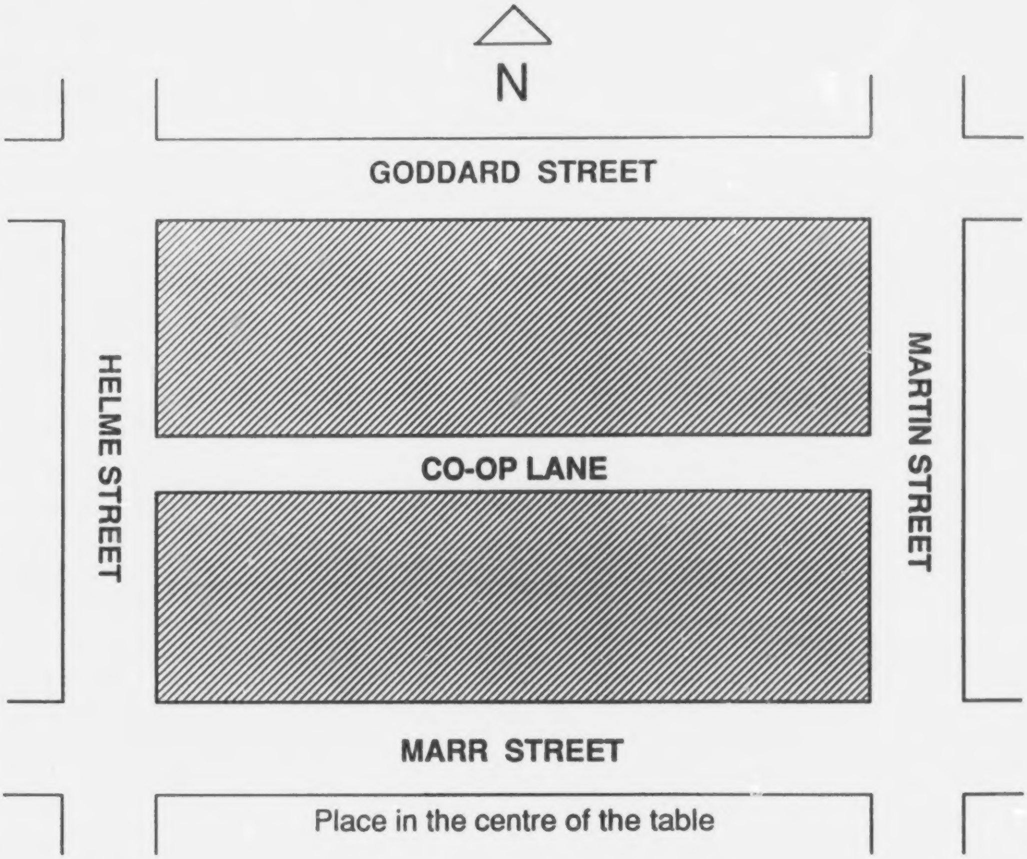
The main points to be made in discussion are:

- Having to read their clues aloud ensures that **everyone has a role to play** in the problem solving process. This is why the problem cannot be 'given away' or read aloud by other people. It also encourages people to **listen to each other**.
- Manipulables (movable pieces) encourage **risk taking** and making guesses in the problem solving process.
- Manipulables are also a technique to keep the action in the centre of the group. Pencil and paper tends to allow all the action to move to one end of the table or to be taken over by one or two people. We now discourage paper and pencil for most cooperative logic problems. (See other points about manipulables in the Activity 4.4 *Problem Solving Stations.)*
- Sometimes it is necessary to discard irrelevant information e.g. *School Fete*. This models real life problem solving situations where important information has to be selected and other facts ignored.
- If participants are interested in using cooperative logics with their students, distribute Handout 4.6G *Publications Which Contain Cooperative Logics.*

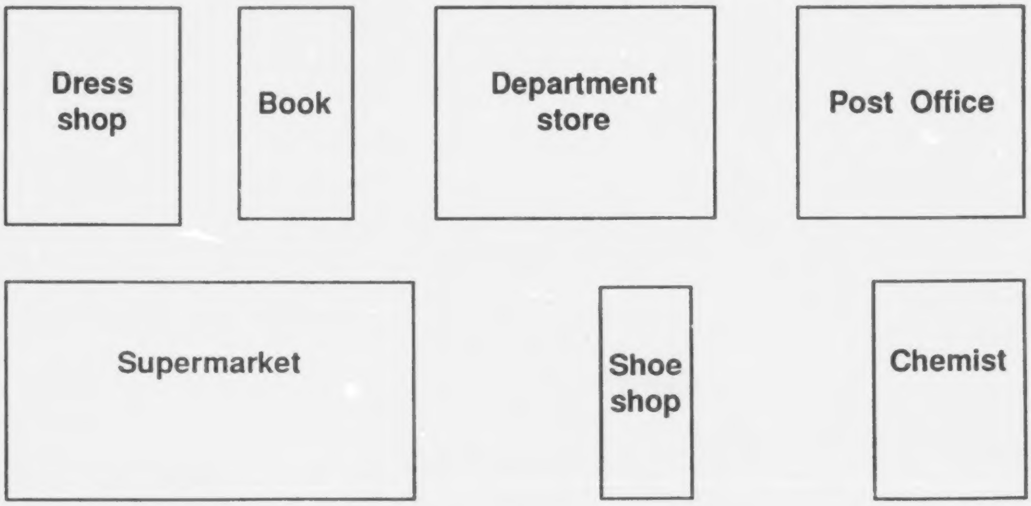
Handout 4.6A p.1

CITY BLOCK

PROBLEM : Arrange the shops in the shaded city block


N
GODDARD STREET
HELME STREET
CO-OP LANE
MARTIN STREET
MARR STREET

Place in the centre of the table


Dress shop
Book
Department store
Post Office
Supermarket
Shoe shop
Chemist

1991 Strength in Numbers: Goddard, Marr, Martin

Handout 4.6A p.2

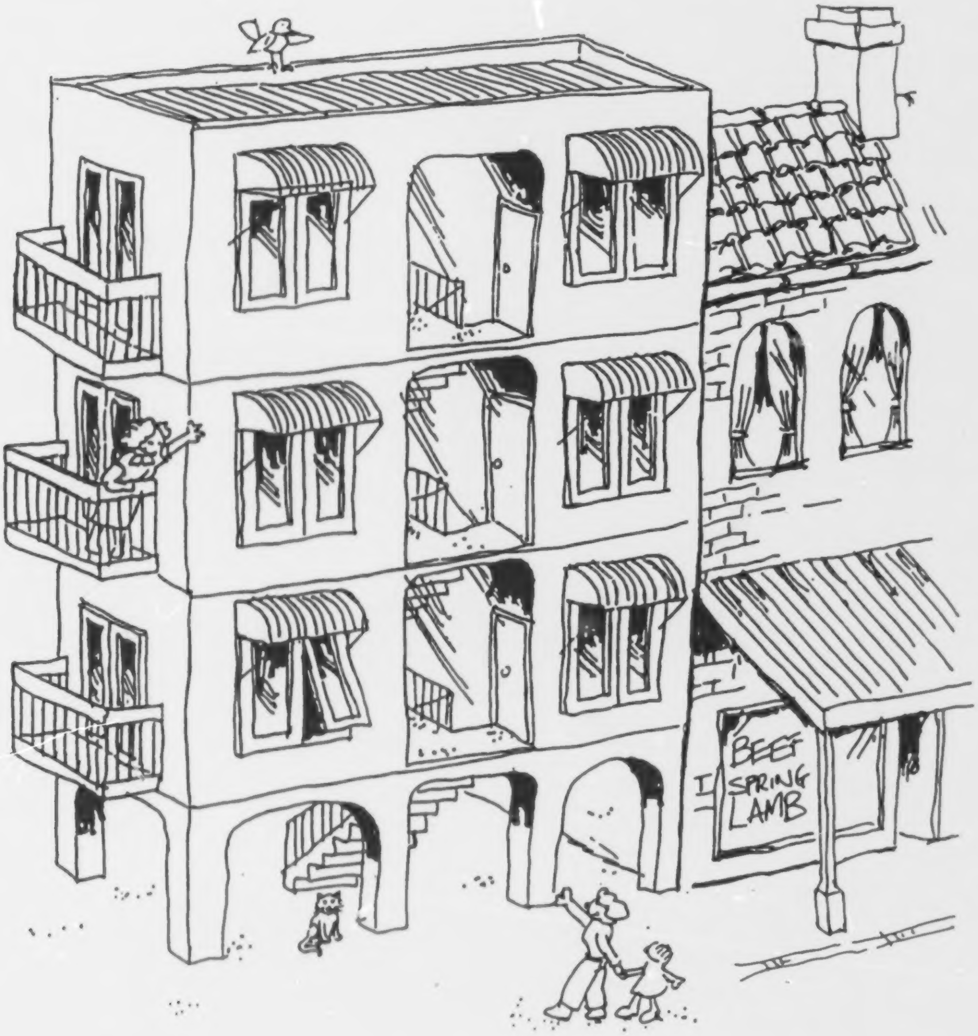
CLUES - CITY BLOCK

Walking from the supermarket to the chemist, you pass the dress shop.	The post office is south of the chemist, and next to the shoe shop.
The department store and the post office are on corners.	The post office is opposite the dress shop.
The chemist shop is east of the supermarket.	The supermarket is north of the bookshop.

1991 Strength in Numbers: Goddard, Marr, Martin

THE FLATS

(Place this in the centre of the table)


BEEF
SPRING
LAMB

Problem: Who lives in which flat?

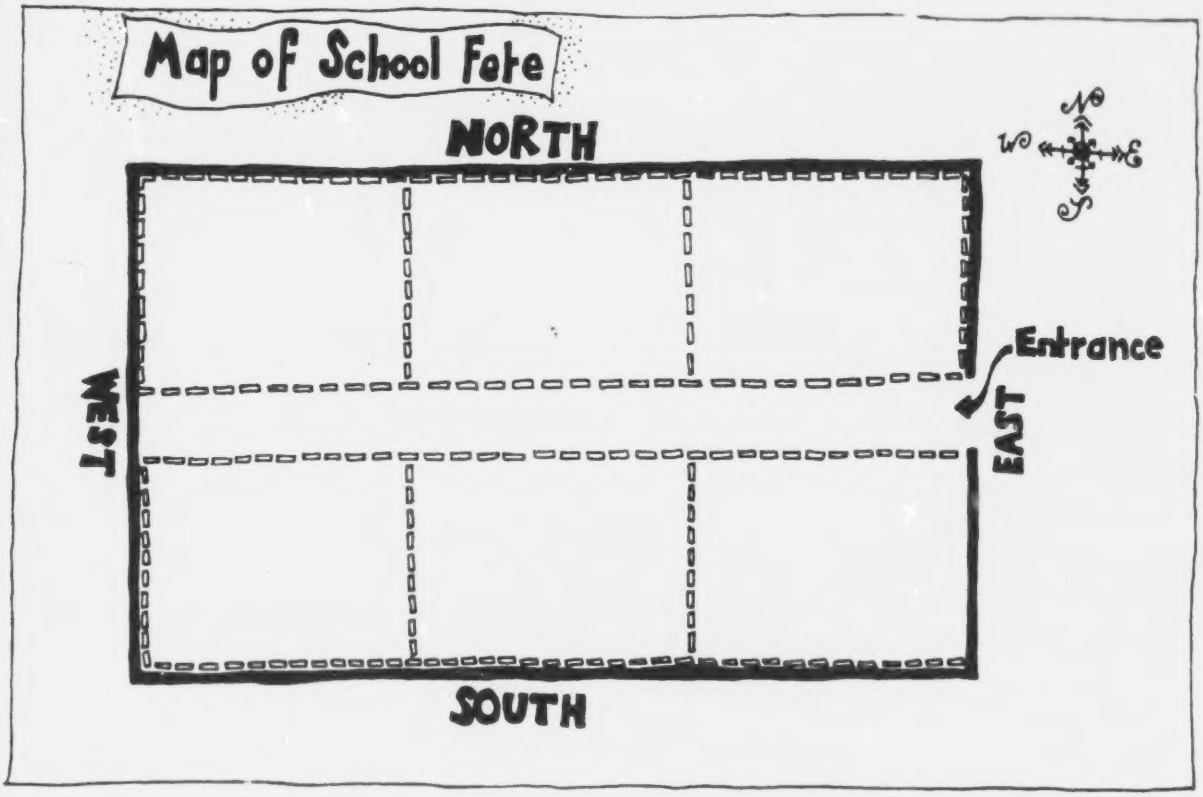
THE FLATS

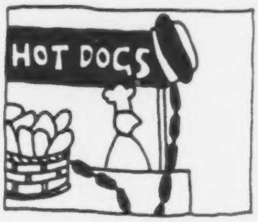
SARTORI	BATES
FISHER	TRAN
JOHANNSEN	WOODS

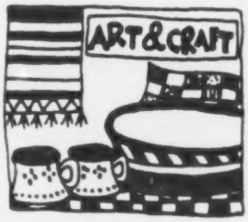
Jo Fisher walks downstairs to feed Maria Sartori's cat when she is away.	The Woods knock on the Tran's floor when their music is too loud.
The Johannsen family hear Mr Wood's feet overhead when he dances.	The Fishers do not live opposite the Johannsen family.
Maria Sartori passes the Bates' flat on the way up to visit the Tran family.	The Fishers grow tomatoes on their balcony in summer.

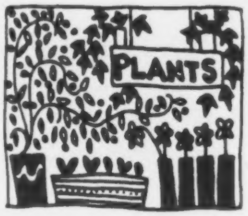
Handout 4.6C p.1

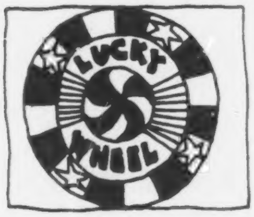
School Fete

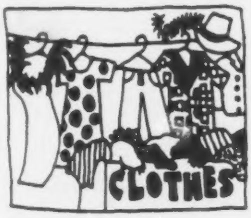

Map of School Fete
NORTH
N
W
E
S
Entrance
WEST
EAST
SOUTH

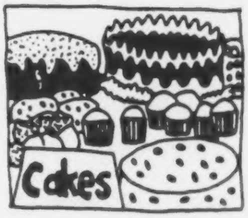







LUCKY WHEEL





Reproduced from *Mathematics: a New Beginning*, B. Marr and S. Helme 1987

School Fete

CLUE CARD: SCHOOL FETE
These clues will help solve the problem.

Read them aloud to the group,
but do not show them to anyone

Problem: Where is each stall at the School Fete?

- The art & craft stall is closer to the entrance than the hot dog stall.
- When Sylvia arrived at the hall, she noticed the clothes stall on her left.

CLUE CARD: SCHOOL FETE
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at the School Fete?

- The plants are west of the art & craft stall.
- The plant stall is opposite the hot dog stall.
- The plant stall made the most profit.

CLUE CARD: SCHOOL FETE
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at the School Fete?

- The hot dog stall is next to the clothes stall.
- Joan donated two dozen lamingtons to the cake stall.
- The art and craft stall was not next to the clothes stall.

CLUE CARD: SCHOOL FETE
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at the School Fete?

- The Annual Fete is held in the school hall every November.
- The clothes are next to the cake stall.
- The lucky wheel is adjacent to the plant stall.

CLUE CARD: SCHOOL FETE *
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at the School Fete?

- When Helen left the hot dog stall she passed the lucky wheel on her way out.
- The cake stall was sold out by lunchtime

CLUE CARD: SCHOOL FETE*
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at theSchool Fete?

- The lucky wheel is north of the clothes stall.
- The first prize on the lucky wheel was a video recorder

*CLUE CARDS marked thus (*) are not necessary for solving the problem. Use these cards if there are more than four players in a group.

Reproduced from *Mathematics:a New Beginning*, B. Marr and S. Helme 1987

Build It : 1

Build It #1

There are six blocks in all.

One of the blocks is yellow.

Build It #1

The green block shares one face with each of the other five blocks.

Build It #1

The two red blocks do not touch each other.

Build It #1

The two blue blocks do not touch each other.

Build It #1

Each red block shares an edge with the yellow block.

Build It #1

Each blue block shares one edge with each of the red blocks.

from *Get It Together* • EQUALS, Lawrence Hall of Science

Handout 4.6E

Build It: 2

Build It #2

There are six blocks in all, in a tower six blocks high.

There is a yellow block on top.

Build It #2

The red block is above the green block.

Build It #2

One of the yellows is above the green block; the other is below it.

Build It #2

Each of the blue blocks shares a face with the green block.

Build It #2

No two blocks of the same color touch each other.

Build It #2

There are two yellows, two blues, one green, and one red in the set of blocks.

from *Get It Together* • EQUALS, Lawrence Hall of Science

Build It: 3

Build It #3

There is a red block directly below a yellow block.

There is a green block on the bottom level.

Build It #3

There is a red block directly on top of a yellow block.

The highest block is on the third level.

Build It #3

There are six blocks in all.

An orange block shares a face with a green block and two others.

Build It #3

A blue block shares a face with a yellow block.

There is a red block on the bottom level.

Build It #3

A blue block touches red and green blocks only along edges.

There are three blocks on the bottom level.

Build It #3

A yellow block touches an orange block only along an edge.

from *Get It Together* • EQUALS, Lawrence Hall of Science

Handout 4.6G

Publications Which Contain Cooperative Logics

Title and Details	Description
Goddard, R. Martin, J. Marr, B. (1991), *Strength in Numbers.* Eastern Metropolitan Council of Further Education and Division of Further Education, Victoria.	A selection of problems with emphasis on essential language and basic numeracy concepts. Language and content aimed at adult literacy and numeracy students.
Marr, B. and Helme, S.*Mathematics: a New Beginning.* State Training Board of Victoria	Contains some Cooperative Logic problems designed for adults but a higher literacy level is required than for *Strength in Numbers.*
(Both available from: Teaching Maths to Women Project, Northern Metropolitan College of TAFE, St Georges Rd Preston Vic. 3072)	
EQUALS (1986) *Get it Together.** Lawrence Hall of Science, University of California.	An entire book of Cooperative Logic problems. They range from simple logic and sorting to higher level maths; especially good for geometry and probability.
Fraser, Sherry et al. (1982) SPACES.* Dale Seymour Publications.	Contains several Cooperative Logic problems at a simple level of problem solving.
New Zealand EQUALS Network, (1989), *Mathematics Teaching: Making Changes: a Handbook for Mathematics Teachers Running Workshops.* *	Includes a series of Cooperative Logic problems using logic and early secondary school standard mathematics.
New Zealand EQUALS Network, (1989), *Towards Better Trigonometry Teaching.**	Contains several introductory and advanced Cooperative Logics using Pythagoras' Theorem and Trigonometric ratios.
(*Available from Objective Learning Materials 41 Martin Pl. Glen Waverley, Vic. 3150)	

Activity 4.7

Exploring Groupwork with Problem Solving

Rationale

Cooperative learning involves more than just putting students together in small groups and giving them a task. In this session we explore that notion by allowing participants time to work through some problem solving tasks in small groups. Participants are then asked to reflect on the dynamics of the group, their own mode of interaction, and feelings about group work.

Working through this process should raise the awareness of typical problems which can emerge in group work, and constructive methods by which they may be overcome.

The session also provides a model which teachers can utilise with their groups of students to enable them to look at their own behaviour as a starting point in the development of improved group working skills.

Aside from the question of group dynamics some interesting problem solving activities are presented to allow participants to practise and develop their own problem solving skills.

Aims

- To raise awareness of the dynamics involved in group work.
- To discuss good models of group work.
- To analyse roles which emerge in groups.
- To examine methods of facilitating effective group work.
- To practise and develop problem solving skills.

Materials and Equipment

- 1 roll of paper streamer for each group (optional)
- Butchers paper (at least 4 sheets for each group)
- Coloured felt pens(at least 2 colours for each group)

Time 1 hour minimum

Handouts

- 4.7A *Self Assessment for Cooperative Groupwork*
- 4.7B *Group Problem Solving Activities for Teachers and Tutors*
- 4.7C *Student Self Assessment for Cooperative Groupwork*
- 4.7D *Reasons for Encouraging Groupwork*
- 2.8A *Problem Solving Guide*

Preparation

Read the background notes to this section, *Interaction and Cooperation* (p 219).

Read the background notes for the *Problem Solving* section of chapter 2 (p.96).

Select **four** problems from those supplied in Handout 4.7B. Each problem is in a separate box to assist with the cutting and layout of your handout. Choose a variety of questions. Try to include numerical, graphical and spatial problems and ensure that they are the most suitable for the level of your participants. Select from problems marked with * to give to the maths teachers initially, to ensure that there is an element of challenge in the task.

Put the **first problem** on a handout by itself and the **other three** on a second handout. Make one copy of each set for each participant.

Photocopy Handout 2.8A *Problem Solving Guide* (p.103) from chapter 2, one for each participant.

Photocopy Handout 4.7C and 4.7D, one for each participant.

Arrange seating with four people around a table, not too far apart. Time taken over appropriate seating is worthwhile. Four is an optimal number, but cannot always be achieved in reality. To obtain maximum participation aim for smaller rather than larger groups.

Conducting the Activity

Grouping the Participants

Divide participants into **groups of four** if possible. If there are maths teachers present as well as literacy teachers it will be preferable to seat them together. Often a maths 'expert' working in a group of literacy teachers can tend to dominate or cause others to feel intimidated merely because of their greater mathematical knowledge. However listening **quietly** to the discussion of people without maths backgrounds and acknowledging their problems and concerns **can** be extremely useful for mathematics trained people.

If the group is a mixture of maths teachers and literacy teachers you may notice when conducting the activity two distinct sets of concerns emerging. Maths teachers may need to spend more time thinking about the groupwork aspects of the session since the ideas and methods may be quite new for them, such as thinking about the importance of language and verbal exchange as a learning strategy. On the other hand literacy teachers may be happy with groupwork ideas and more concerned with the problem solving strategies.

The First Problem

Ask participants to clear the table tops completely of notes before you start. Distribute coloured textas and butchers' paper.

Hand out a copy of the first problem to each participant. Tell them they have 10 minutes to solve the problem and ask each **group** to come up with an agreed solution to the problem. **Do not** mention group dynamics at all at this stage.

Two minutes before you wish to complete this activity give a gentle warning, such as:

> *I'll be asking you to finish in two minutes and would like you to have agreed on your answer by then.*

Thinking about Group Dynamics

As the groups finish, distribute Handout 4.7A *Self Assessment for Cooperative Group Work*, one to each participant.

Ask participants to take a minute or two to fill the self assessment form in **by themselves** without prior discussion. Then, when the forms have been filled in, allow a few minutes for informal discussion within each group. This will probably focus on impressions in the **About the Group** section.

Solution to the Problem

Participants will be eager to verify the answers to their problem solving task. So, before proceeding with a discussion of groupwork, compare solutions and the methods used by each group to obtain them. Refer to Handout 2.8A *Problem Solving Guide* to give names to strategies used. This will be given to participants later in the session.

Discussion of Groupwork

Now is the time to introduce some of the ideas about the desirability of encouraging group work. The main points are described in Handout 4.7D and should be discussed by the group at this stage.

Continuing the Problem Solving

Before proceeding to further group activities, ask everyone to **reflect carefully** on what they wrote on their *Self Assessment for Cooperative Groupwork* form and to keep it in mind as they participate in solving the next few group problems.

Continue the session by distributing the other three selected problem solving tasks and asking the groups to spend 8 to 10 minutes on each one. Groups who are waiting for others to finish could discuss alternative methods or extensions of the problems.

When time has elapsed bring the entire group together for discussion.

Discussion

Discussion should focus on two aspects: Problem Solving Strategies and Group Dynamics. If possible, tackle group dynamics first.

Discussion of Group Dynamics

Ask participants to look at their *Self Assessment for Cooperative Group Work* which they had filled in after the first activity.

This time they should mark with an **asterisk** any of the statements for which **some change occurred** during the last few activities.

For example, you initially may not have ticked the sentence 'I listened to other people's ideas', but you may have begun to do this more over the latter problems, so put an asterisk.

Allow 5 minutes for reflection and informal exchange on this.

Now round up this discussion on group dynamics, covering as many as possible of the points made in the background notes: *Interaction and Cooperation.* Possible discussion starters are as follows:

- *Did stopping to* ***reflect*** *on group dynamics in any way* **alter** *your subsequent behaviour or that of the small group?*
- *Could we use similar strategies to make* **students** *aware of their interactions?*
- *Did being in groups make it easier to tackle more* **complex problems***?*
- *What* ***roles*** *or typical behaviours, might you expect to find emerging in groups of students?* (At this point it is wise to steer discussion away from the particular groups involved in today's activities.)
- *Which of these behaviours would present* **problems** *to effective group work?*
- *What* ***strategies*** *might you adopt to get over these problems with your students?*

Inform the group that for students whose language is not sufficiently sophisticated to cope with Handout 4.7A *Self Assessment for Cooperative Group Work* that is used in this session we have included a possible student version, Handout 4.7C *Student Self Assessment for Cooperative Groupwork.*

Distribute this to participants and briefly discuss its use: if students have insufficient levels of reading and writing it could be used in a verbal mode, either as a whole group, taking turns to respond to Question 1 then Question 2 etc, or one to one with the teacher.

Discussion of Problem Solving Strategies

Treat each problem separately. If the group is fairly homogenous, ask small groups to report on their solutions and strategies. If some groups were tackling more advanced problems you may prefer to hand them the solution to peruse and discuss amongst themselves or for open ended problems go from group to group and hear their solutions.

Notes on the Problems

On shapes and sizes

Some startling discoveries can be made when solving this problem, especially by people with little maths experience. It is a good one for gaining familiarity with the language and concepts of area and perimeter. It also illustrates the use of diagrams or manipulables in solving problems.

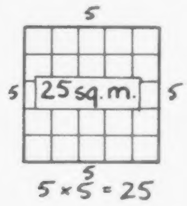
Statement 1

Drawing pictures or using cardboard or wooden tiles should show that **this statement is false.** The area becomes four times as big.

Statement 2

Yes, it is true. You can find the perimeter because the square root gives the side length and the perimeter is the sum of four of these sides.

E.g. If a square has an area of 25 square metres then each side must be 5 metres,



so the perimeter will be 5+5+5+5=20 metres.

Statement 3

Not true: Many rectangles can have the same perimeter but be very different in area. Ask participants how many rectangles they can draw with a perimeter of 20 cm, for instance. E.g.(1 by 9 - area 9 sq. cm) (2 by 8 - area 16 sq. cm)

Statement 4

Imagine light from a torch. Obviously as you put a paper closer to the torch the ring of light becomes smaller (but brighter). **The statement is false.**

Odds and Evens

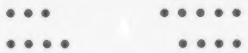
This problem can be tackled on two levels: one, making rules based on many tested examples; two, actually finding methods of proof either by diagram or more sophisticated mathematical processes.

One possible model for people with little maths background utilises the 'shape' of the numbers.

Even numbers can be thought of as different length rows of pairs, such as

• • •   or   • • • • •
• • •   • • • • •

Odd numbers can be thought of as even numbers with a spare single on the end such as



Statement 1 Two even rectangles added or placed together will make an even longer row of pairs, obviously still even, so **the first statement is true**.

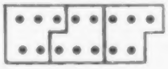
Statement 2 If two of the odd numbers are joined then their spare singles will combine to make a pair, and the result is a row of pairs so the **this statement is true.**

Statements about multiplication might be illustrated similarly by thinking of multiplying as adding a number of 'lots of' e.g 6 x 4 is six lots of four added together.

Statement 3 Any number of even numbers combined will remain as rows of pairs - **so statement 3 is true.**

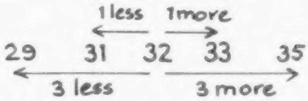
Statement 4 By experimenting with combining an odd number of lots of a general odd number shape it can be easy to see that there will always be a spare single on the end. So **statement 4 is true**.

E.g. 3 x 5



The Averages questions

These should help to clarify the concept of an average. Obviously there are no **right** answers but hopefully the ideas of some measurements being higher and some lower than the average will be discussed, e.g. ages could be combinations of pairs either side of the average.



The Nines Problem

This seems true, and fascinating at first

e.g. 9 x 5 = 45 ----------> 4 + 5 = 9

9 x 12 = 108 ----------> 1 + 0 + 8 = 9

but consider larger numbers such as

9 x 51 = 459 ---------> 4 + 5 + 9 = 18

However if we look at 18 we see that 1 + 8 = 9

Eventually if we keep adding we reach 9. Try some larger numbers.

Some participants may come up with theories of 'Why is it so?' but keep it brief-don't let others in the group be made confused by lengthy explanations

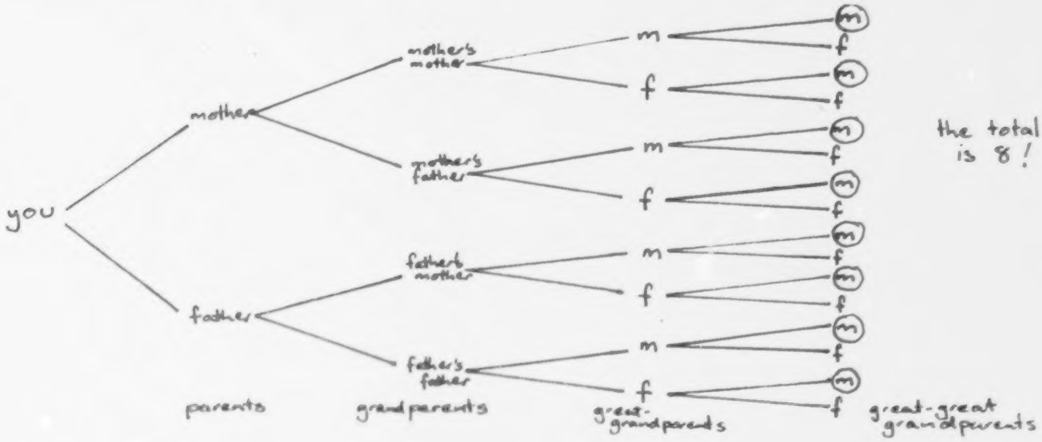
Painted Tyres

There seem to be two possibilities that people find attractive. One conjecture is that a series of marks appear one tyre circumference apart. The second conjecture is that two series of marks appear, one for the front tyre and one for the back. This latter conjecture is sometimes qualified by observing that it depends on the distance between the bicycle wheels.

If participants decide that the tyres made separate marks discuss what would happen if two people on unicycles came by a few minutes apart Altering the conditions often shows what matters. **In this case, as long as the wheels have the same radius, they will deposit marks in the same spots - one series of marks.**

Grandmothers

One solution of this is via a classic strategy in mathematics - the tree diagram.


you
mother
father
mother's mother
mother's father
father's mother
father's father
m
f
m
f
m
f
m
f
m
f
m
f
m
f
m
f
m
f
m
f
m
f
m
f
parents
grandparents
great-grandparents
great-great grandparents

the total is 8!

The Common Numbers and What Shape? Questions.

These are open ended so no answers are supplied. These questions may be useful to clarify some mathematical language such as prime numbers (those with no factors), equilateral triangles (3 equal sides) for participants with little maths background.

The Graph Questions

There are no exactly right answers, but groups should justify their selection or stories with reference to particular features of the graph. Ensure they notice features such as: faster speeds are indicated by steeper slopes or gradients; and stopping, by horizontal patches.

Self Assessment for Cooperative Group Work

Tick the statements that were true for you in this group activity.

About Myself

I spoke in the group.

I listened to other peoples ideas.

I suggested an idea to the group.

I encouraged other people to contribute.

I felt comfortable working in the group.

Working in the group made me feel anxious.

Working in the group made the task easier.

I would have preferred to do this task alone.

I would have preferred to do this task with just one other person.

About the Group

Everybody contributed to the task.

One or two people tended to dominate the discussion.

Some people in the group just did their own thing.

People could ask the group for help if they didn't understand.

Group Problem Solving Activities for Teachers and Tutors

On Shapes and Sizes

For each of the following statements, decide whether you think the statement is **always** true, **sometimes** true, or **never** true.

- **If I double both sides of a square its area will also double.**
- **If I know the area of a square I can find its perimeter.**
- **Two rectangles with the same perimeter will have the same area as well.**
- **If a slide projector is moved closer to a screen, the picture on the screen will be larger.**

Odds and Evens

For each of the following statements, decide whether you think the statement is **always** true, **sometimes** true, or **never** true.

- **If two even numbers are added together the result (sum) is even.**
- **If two even numbers are multiplied together the result (product) is even.**
- **If two odd numbers are added together the sum is odd.**
- **If two odd numbers are multiplied together the product is odd.**

Average Age

32 is the average age of a group of 5 students.

What ages could they be ?

Average Height

Use the streamer provided to find and display the average height of your group.

The Nines Problem

*

Is this statement **always** true, **sometimes** true, or **never** true?

If a number is divisible by 9 then the sum of its digits is 9.

Painted Tyres

*

Once while riding on my bicycle along a path I crossed a strip of wet paint about 15 centimetres wide. After riding a short time in a straight line I looked back at the marks on the pavement left by the wet paint picked up on my tyres. What did I see?

Grandmothers

If all your great great grandmothers were still alive, how many would there be?

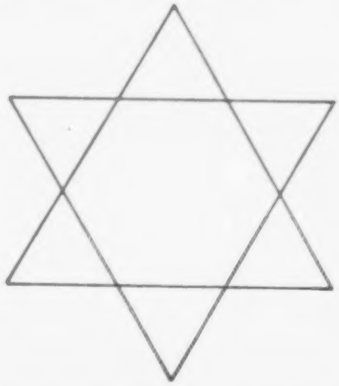
Common Numbers

What things do all of these numbers have in common?

11 13 17 19

What Shape?

Describe this shape in as many ways as you can.



Coming to College

Helen: I had just left home when I realised that I have aerobics tonight and I'd forgotten my gear! So I went back home and then I had to hurry to be on time.

Yvonne: I always start off very calmly. After a while I speed up , because I don't like to be late.

Franca: I went by car this morning, but ran out of petrol a few blocks from the college. So I had to walk for the rest of way and was just on time.

Kathy: I put my daughter Lisa onto her special seat on my bike and rode to the childcare centre. I talked for a while to the new worker there and then hurried to the college.

Mary: . ...

....

....

....

....

....

....

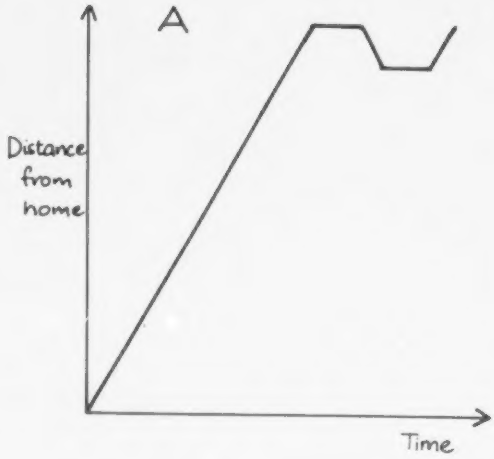
....

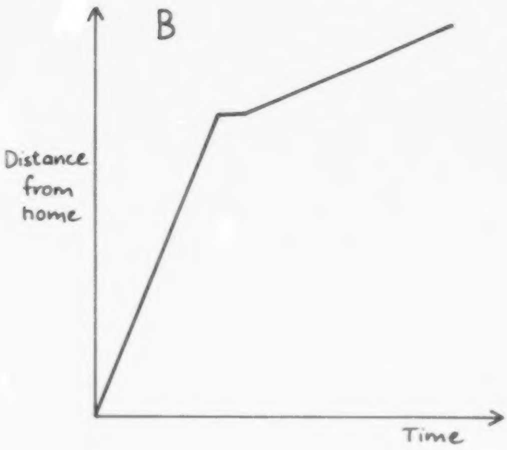
Coming to College

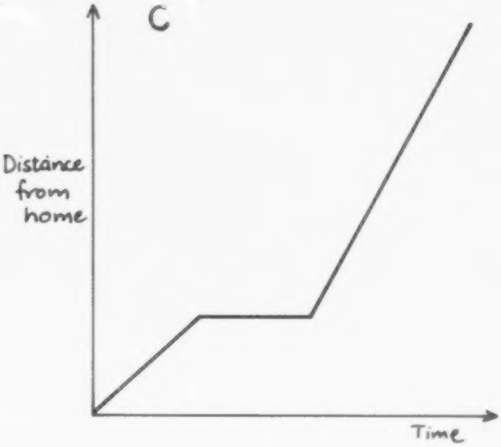
Students who go to College at Midtown usually travel by bicycle.

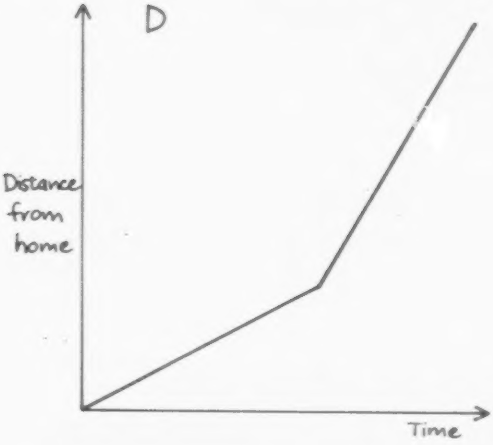
Here you can see five graphs. On the following page there are four stories. Which story goes with which graph?

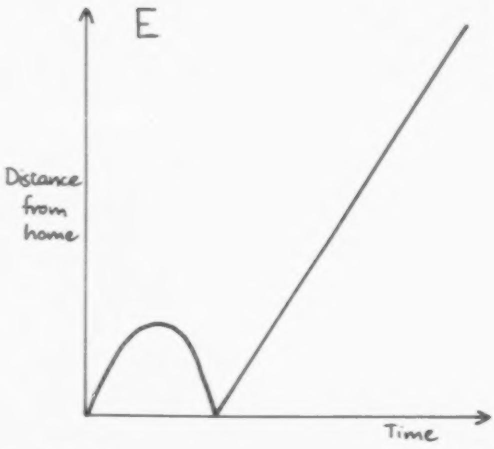
What do you think Mary might have said? (Fill in the spaces on page 2 of this worksheet.)





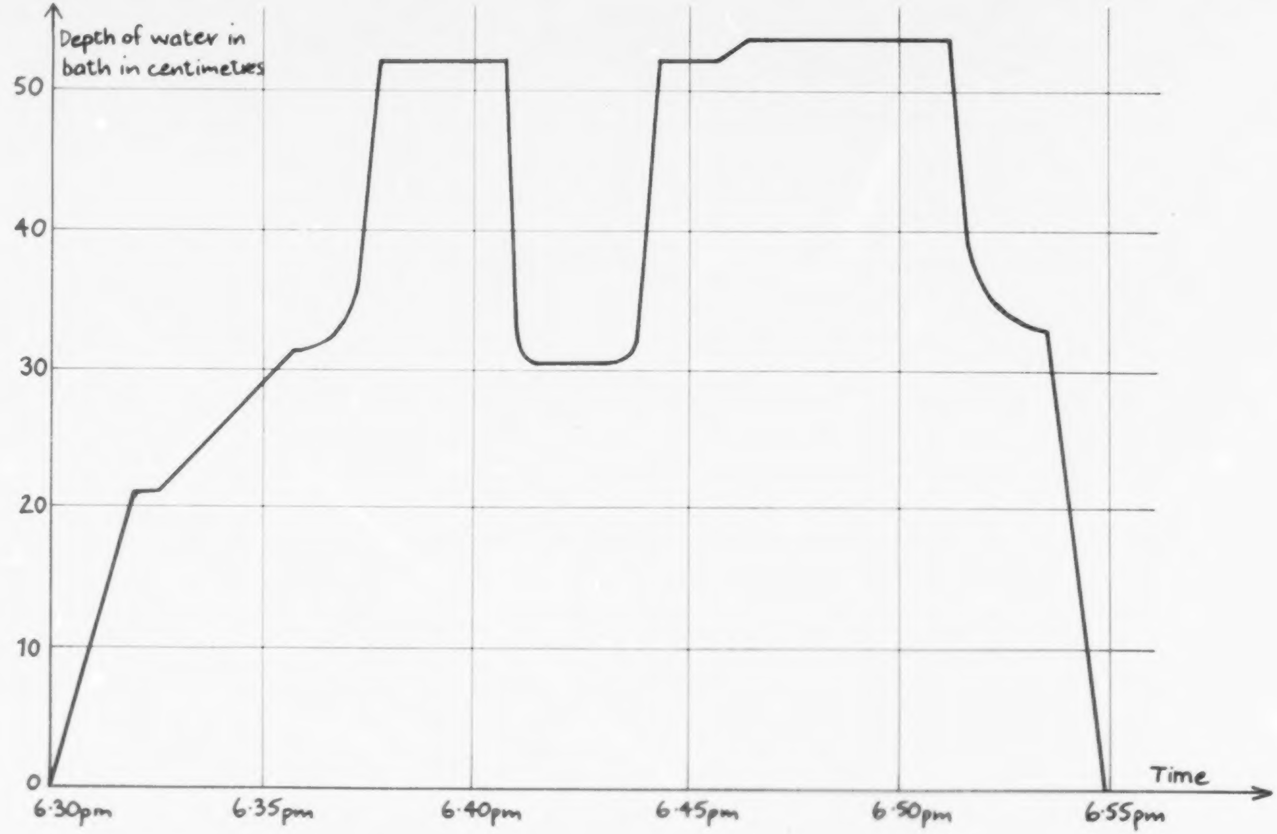






The Bath

Susie came home from a very tiring day at work and decided she needed a relaxing bath. The graph below tells part of the story. Can your group put the story of Susie's bath into words ?

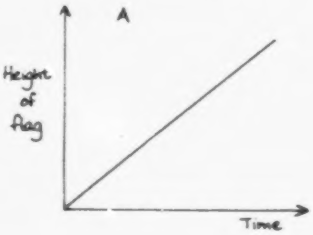


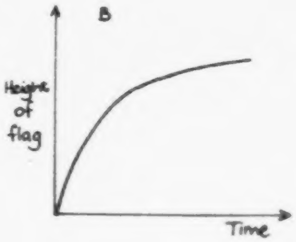
Breaking The Maths Barrier, B. Marr and S. Helme, 1991

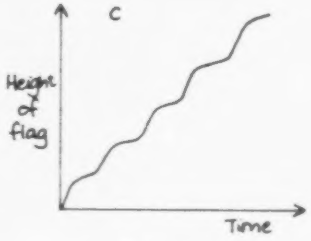
The Flag

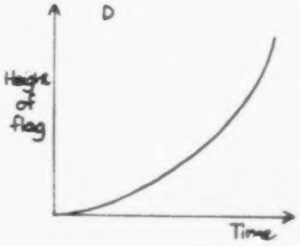
Every morning at the summer camp, the youngest girl guide has to hoist a flag to the top of the flagpole.

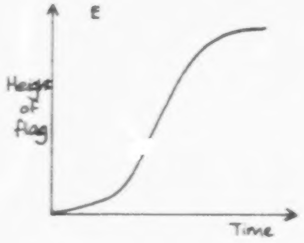
Which graph is the most likely to describe the situation as the Girl Guide hoists the flag ?

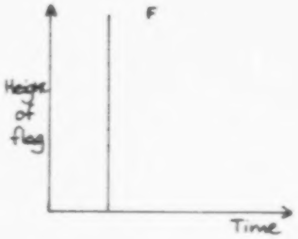












If you think that none of these graphs are realistic, draw your own version and explain it.

from *Mathematics Teaching: Making Changes: A Handbook for Mathematics Teachers Running Workshops.* New Zealand EQUALS Network

Student Self Assessment for Cooperative Group Work

Tick the statements that were true for you.

I liked being in the group.

I listened to others.

I learned something.

I talked in the group.

I talked too much.

I helped other people.

I didn't talk much.

Reasons for Encouraging Groupwork

Not only is groupwork an effective learning strategy (i.e. students learn better when some group work is incorporated into their course) but all people have to function in groups at some time in their lives. This may be in work teams, committees, parent clubs, political meetings, or merely organising a social event. In all such situations interpersonal skills are important. Some of these skills are:

- listening to others in a sensitive way
- putting forward your ideas in an effective manner
- encouraging others to participate
- reaching consensus in a group.
- resolving conflict
- giving and receiving constructive feedback

as well as the more practical skills of:

- taking notes
- summarising discussions
- reporting your findings to others

Using Games in Adult Numeracy

Games are a very valuable resource in teaching numeracy, providing an enjoyable means for students to practise and develop their skills. Although you may expect adults to think games are childish and irrelevant, most respond enthusiastically and appreciate their educational and social value.

Why Use Games?

Games can contribute in many ways to a numeracy class, such as:

- **breaking the ice with students who are new to each other**. A game can be used to get a new group of students talking and working together, and to quickly introduce a new student into an existing group.
- **encouraging student interaction.** Students involved in a game will spontaneously engage in discussion about the rules, their discoveries and their strategies and the questioning and justifying of their play to one another.
- **developing new skills through discovery**. A game can provide a light hearted and non-threatening context in which to learn new skills or discover new ideas or information, and a welcome alternative to teacher-centred instruction.
- **consolidating skills in an enjoyable way.** Practice can be tedious when presented as pages of repetitive exercises, but in the context of a game, students motivated to win will play again and again, reinforcing their skills without getting bored.
- **providing a warm up at the beginning of the class**, either as a means of diverting students attention away from other concerns or to energise those who come to a night class at the end of a long day.
- **providing an enjoyable break from the normal routine.**
- **providing an interactive activity at the end of the class,** so that the class concludes on a positive note.
- **introducing aspects of Australian culture** to students who are unfamiliar with the card and board games which are very popular in this country.

A good place to start is to choose a favourite game of yours or some of the students.in the class. Play it together and discuss the skills which students are practising whilst playing it.

What Type of Games are Suitable?

Suitable games include card games, board games, commercial games or games invented by the teachers or students. Even a standard card game such as Pontoon or Twenty-one contains the valuable numeracy skills of number recognition and addition, and develops an intuitive understanding of probability. Other games have been specially developed to teach or reinforce particular numeracy skills, for example *Multidigit* (Activity 4.9) which reinforces and explores place value.

You can also encourage students to demonstrate games they know. This provides an opportunity for them to practise their communication skills - for instance, when they are explaining the rules - and you may see an opportunity to adapt a game they enjoy to include some of the skills they are learning in class. Most importantly, introduce games with a confident attitude, and students will give them a go.

What Games Should I Use?

There is a wide range of card games, board games, dice games and activities available from educational publications, and educational and commercial suppliers. Here are a few guidelines:

- It is not necessary to use expensive or complicated games for a good result in the classroom. Often the simplest games work extremely well.
- Choose games that do not take too long to play. Students can lose interest if there is no winner after a short time. It is better to play several rounds of a simple game than one or two rounds of a more complex game. This allows students more chance of winning a round and maintains their concentration.
- Make sure that the games that you choose have strong element of chance, so that those who have the most skills do not necessarily win all the time.
- Choose games with simple rules, otherwise students spend all their time and energy trying to understand and remember the rules and lose sight of the skills which the game is intended to develop.

The Activities

Three games are presented in detail to use with your staff development group

Activity 4.8: ***How Close Can You Get ?*** is adapted from a card game in *Family Math* which uses a standard deck of cards to practise arithmetic skills.

Activity 4.9: ***Multidigit*** explores place value and develops skills in writing and verbalising large numbers.

Activity 4.12: ***Digititis*** develops understanding of mathematical language and terminology.

Following the Activities

At the conclusion of the activities you could round up and consolidate with a discussion by considering these questions:

- *What value do games have in a numeracy class?*
- *When and how should they be used?* (You may wish to refer to the introductory notes to assist the flow of ideas and/or hand out copies of these notes to participants.)
- *What other games might be used or adapted for a numeracy class?* Collect and record participants' ideas.
- Distribute Handout 4.8A *Ideas for Numeracy Games*

Activity 4.8 How Close Can You Get?

Rationale

A standard deck of cards is a cheap, easily accessible yet highly valuable resource for developing and practising number skills such as addition, subtraction, estimation and mental arithmetic. This game presents one example of using cards to learn and practise arithmetic facts, and acts as a stimulus to participants to think creatively about adapting and creating their own card games.

Aims

- To develop awareness of alternative approaches to learning and practising number skills.
- To encourage participants to develop their own card games.

Materials and Equipment

- Packs of cards (1 per group of 4)
- Pencils and paper

Time

20 minutes

Handouts

- Handout 4.8A *Ideas for Numeracy Games*

Preparation

Photocopy Handout 4.8A *Ideas for Numeracy Games*, one copy for each participant.

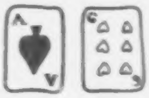
Conducting the Activity

Remove the tens and picture cards from a deck of cards, leaving ace through to nine for the game. Shuffle the cards.

Deal each player four cards, face down.

Turn up two more cards, and place one in the ten's place and the other second in the one's place to form **the target number.**

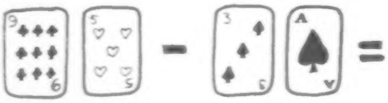
For example, a six then an ace makes 61.



Round 1

Now the players turn up their four cards and arrange them into **two** 2 digit numbers so that when one number is **subtracted** from the other, the result will be as close to the target number as possible.

For example, if a player has Ace, 5, 3, 9, the best he or she could do is 95 - 31= 64.



Their score for that round would be the difference between their result and the target number, i.e. 64 - 61 = 3.

Note that the result of this subtraction can be either over or under the target number.

Play a few rounds (usually three is enough for demonstration purposes) dealing new cards for each round.

The winner is the player with the lowest total score after the agreed number of rounds.

Note: You may wish to omit scoring until the players feel comfortable with the game.

Extensions

Play as a group of four or five, working towards a low group score, competing with other groups at other tables.

Allow players to trade cards.

Try a 3 digit target number and six cards for each player.

(This game is adapted with permission from *Family Math*, which contains many other numeracy games and activities.)

Discussion

Ask for feedback on the game. Participants may comment that the game is too complex for basic numeracy students. If so, ask for suggestions on simplifying the game, and record them.

Ask participants to spend five to ten minutes in their groups discussing further games they could use or develop using a deck of cards.

Bring the group together and record their ideas.

Distribute Handout 4.8A, which contains further ideas for numeracy games. (Alternatively, wait until the completion of all the activities in this section.)

Ideas for Numeracy Games

Publications

Education Department of Victoria, (1985), *Guidelines in Number*. Melbourne.

Goddard, R. Marr, B. & Martin, J. (1991), *Strength in Numbers*. Eastern Metropolitan Council of Further Education and Division of Further Education, Victoria.

Kerr-Stenmark, J.K. et al. (1986), *Family Math.* Lawrence Hall of Science, University of California

Marr, B. and Helme, S. (Eds 1987), *Mathematics: A New Beginning*. State Training Board of Victoria. (Available from Mathematics Dept., Northern Metropolitan College of TAFE, St. Georges Rd, Preston Victoria 3072)

Silvey, L. (1978), *Polyhedra Dice Games*. Creative Publications, Palo Alto, California.

Commercial Games

- UNO - card game
- STUN - card game
- Equable
- Number Boggle
- Dominoes - (normal type as well as ones which emphasise particular skills e.g. Decimal dominoes, Metric dominoes.)
- Yahtzee - number version
- Mastermind
- Tangrams (and other shape games)
- Card games using a standard pack (e.g. Pontoon or '21')

Activity 4.9 **Multidigit**

Rationale

This is an enjoyable way to develop an appreciation of place value for digits in large numbers. Students also practise reading aloud and writing large numbers in words. Although a knowledge of place value is important in developing a winning strategy, there is still a significant element of chance.

Aims

To introduce a game which develops an awareness of the importance of place value.

To use an enjoyable activity as a basis for learning to express large numbers verbally and in written form.

Materials and Equipment

- One ten sided dice

Handouts

- 4.9A *Multidigit* (1 per participant)

Time 10 minutes

Preparation

Photocopy Handout 4.9A *Multidigit*, one for each participant.

Conducting the Activity

Participants play individually. Give one grid to each player and start the game with the smaller grid.

The aim of the game is to obtain the highest possible total at the end of each round.

Each time the dice is rolled the player puts the number shown into one square of the grid according to the following procedure:

- The first roll of the 10 sided dice gives a number for row 1 .Players enter this number in the only box in row 1 of the grid.
- The dice is rolled again for the second row. There are two boxes in row 2 and the players must choose which box they will place the number in. If for example the number was 6, in the first box it will be worth 60, in the second it will be worth 6. Remember the aim is to get the highest number so they must decide whether to take the 60 hoping that the next number thrown is smaller than 6.
- Roll the dice the second time for row 2. If for example it is 4, this number put in the second box to give the number 64.

- Proceed in this way completing each row before moving on to the next, until all the boxes in the grid are filled. Players must fill a box each time the dice is rolled.

Players now add together all the numbers as in normal addition of numbers set out in columns and find their total.

The next step is to encourage the players to read their numbers out loud. This is best done by a series of questions such as:

- *Who thinks they might have the biggest number? What is it?*
- *Does anyone have a higher number?*
- *Does anyone have a number between these?*
- *Who thinks they have the lowest number?*

The players should be reading the numbers out in full i.e.6578 would be 'six thousand, five hundred and seventy-eight.' While listening to others reading their numbers,players should decide whether their own is bigger or smaller.

Once you have practised with the small grid, move on and repeat the game on the bigger grid where large numbers in the millions will be encountered.

Discussion

Ask for feedback - *what did you like about the game*?

Discuss how the game can be used most effectively in a numeracy class, using the following questions as a guide:

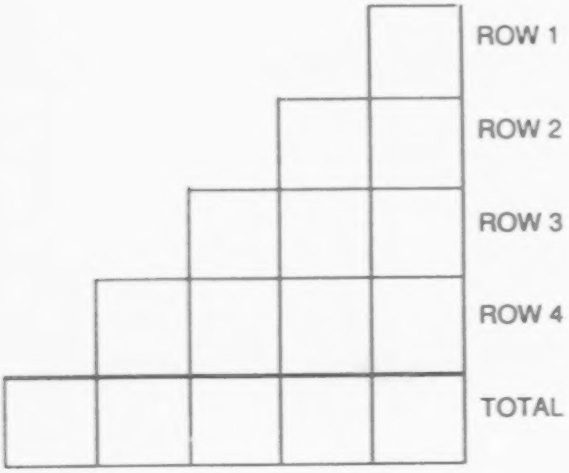
- *How do you ensure that all students read their numbers out loud?*
- *What strategies would you use for students unable to read the numbers out loud?* (E.g. record numbers on board and then practice reading them out loud.)
- *How would you adapt the game for students for whom the jump in grid sizes is too rapid?* (E.g. use a more gradual increase in grid sizes.)

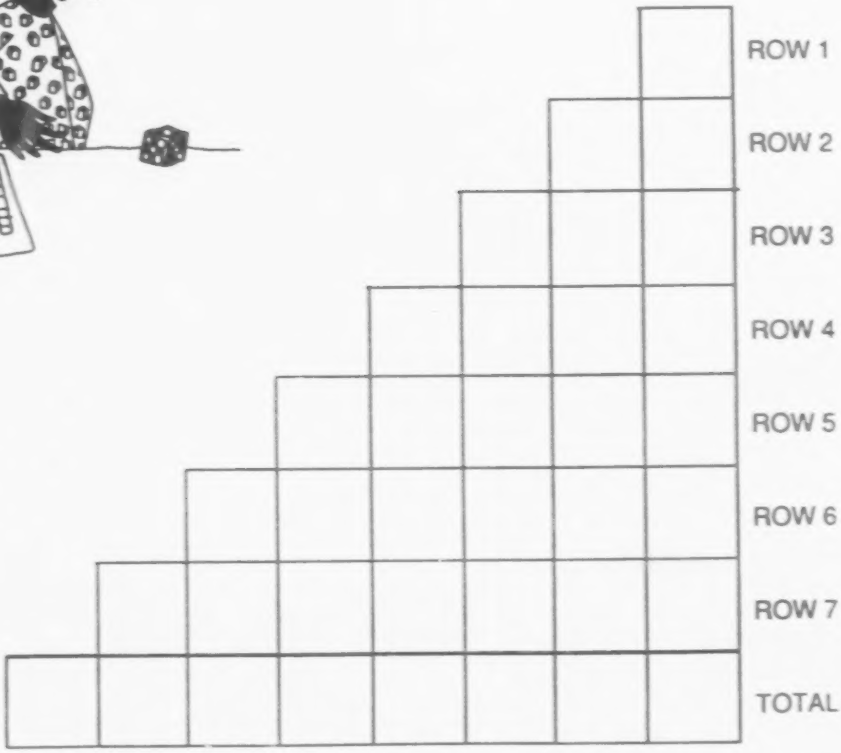
Ask participants for ideas and extensions and variations to the game. These could include:

- Asking students to stand and form a line in order of their numbers, from highest to lowest.
- Students writing a cheque for the number they obtained, to practise writing their number in words.
- Encourage students to get a sense of the size of their numbers by asking them to make a 'wish list' of what they could buy with the amount of money in their total.
- Students rounding their number to the nearest thousand, hundreds of thousands and million.
- Changing the aim of the game to obtaining the lowest possible total.

Handout 4.9A

MULTI DIGIT Worksheet


ROW 1
ROW 2
ROW 3
ROW 4
TOTAL


ROW 1
ROW 2
ROW 3
ROW 4
ROW 5
ROW 6
ROW 7
TOTAL

1991 Strength in Numbers: Goddard, Marr, Martin

Activity 4.10 **Digititis**

Rationale

This game involves using and understanding mathematical language. Although competitive, winning is based purely on chance, not on mathematical skills and abilities.

Aims

To explore a game in which players practise using and understanding mathematical language and basic operations.

Materials and Equipment

- Pencils and paper

Time 10 minutes

Handouts

- Handout 4.10A *Digititis*
- Handout 4.10B *Digititis for Beginners*

Preparation

Photocopy Handout 4.10B *Digititis for Beginners*, one for each participant. Although Handout 4.10A *Digititis* is probably too advanced for basic numeracy students, some participants may like to have a copy, so have copies available just in case.

Conducting the Activity

Make it clear to the participants that this version of the game is designed to challenge **them** and that a simpler version of the game designed for basic numeracy students will be provided.

Ask participants to write down three numbers less than ten (e.g. 1, 4, 7).

Read out the questions on Handout 4.10A *Digititis*, and ask participants to record 'yes' or 'no' to each question.

Encourage participants to clarify the instructions with each other if they need to.

The winner is the one with the greatest number of 'yes' answers.

Discussion

Ask for feedback on the game:

- *What skills are embedded in this activity?*
- *Was it enjoyable? If so, why?*

Distribute copies of Handout 4.10B *Digititis for Beginners* and ask for comments. Stress that it is important to use language that is appropriate for each individual group of students. Students should also be encouraged to talk to each other and help each other on the task, because the main goal of the activity is practising and understanding mathematical language.

At the conclusion of the session, you may like to distribute copies of Handout 4.10B *Digititis* to participants who feel they can use it.

Digititis

Read out the questions, asking players to record a 'yes' or 'no' answer to each. When all questions have been read out, ask players to count their 'yes' answers. The winner is the one with the most 'yes' answers.

Note: The instructions in the brackets provide an alternative explanation for players who ask for further clarification.

1. Is the third number the largest ?
2. Is the middle number the smallest?
3. Is the difference between the largest and smallest 5 or more? (Subtract - or take away - the smallest number from the largest number. Is the answer 5 or bigger?)
4. Is the sum of the three numbers even? (Add the three numbers together. Is the answer an even number?)
5. Is the sum of the three numbers 15 or more? (Add the three numbers together. Is the answer 15 or more?)
6. Is the sum of the three numbers divisible by three? (Add the three numbers together. Will three go exactly into the answer? or If you divide the answer by 3, is there nothing left over?)
7. Is the product of the two smallest numbers less than 20? (Multiply the two smallest numbers together. Is the answer less than 20?)
8. Is the product of the two largest numbers more than 40? (Multiply the two largest numbers together. Is the answer more than 40?)
9. Can you make a three digit number with your three numbers that is smaller than 329? (A number between 100 and 999 is a three digit number.)
10. Can you write a mathematical expression using all of your three numbers and any of '+', '-', 'x' or '÷' to get either 4 or 5 as an answer? (Can you write a sum using each of your three numbers once and once only and any of '+', '-','x' or '÷', to get an answer of either 4 or 5?)

Extension

Can you find the three numbers that would give 'yes' to all ten questions?

Variation

Decide the winner on any number of 'yes' answers (not necessarily the most).

This activity was kindly provided by the Family Maths Project Australia (FAMPA).

Digititis for Beginners

Instructions

Before starting the game, discuss with students the meaning of *digit, product, sum, difference even* and *odd.*

Ask players to think of two numbers between 0 and 9 and write them next to each other, to make a two digit number. For example, 3,7.

Read out the questions, asking players to record 'yes' or 'no' to each. When all questions have been read out, ask players to count their 'yes' answers.

The winner is the player with the most 'yes' answers.

Questions

1. Is the first number even?
2. Is the second number odd?
3. Is the first number bigger than the second number?
4. Do the two numbers add up to more than 7?
5. Is the product of the two numbers even?
6. Is the product of the two numbers more than 20?
7. Is the sum of the digits odd?
8. Is the sum of the digits less than 15?
9. Is the product of the digits less than 30?
10. Is the difference between the digits 3 or more?

Extension

Can you find a two digit number that would give 'yes' to all 10 questions?

Meeting Individual Needs

Dealing with individual differences is an area of great concern for literacy and numeracy teachers, and a problem even for the most experienced teachers. It is an unavoidable reality in adult basic education that students come to numeracy classes from a wide range of backgrounds, with different interests, needs and abilities. This section addresses this issue, and suggests strategies we can use to cater for individual differences.

How Do Students Differ From Each Other ?

There are a number of factors which contribute to differences between students. These include :

1. Background

Students' social, cultural, political and ethnic backgrounds can vary significantly and influence students' expectations of the content of the course, their role as students and the role of the teacher. For instance, students more familiar with rote learning may find it hard to adapt to modern teaching methods. Some students may find it difficult to relate to the teaching materials used in class as many teaching resources still show evidence of cultural, gender, class or racial bias.

Differences in students' language backgrounds or value systems can also contribute to communication difficulties and make the task of producing a cohesive and cooperative classroom atmosphere particularly challenging.

2. Skills

The wide range of skills encountered in numeracy, reading, written and oral communication complicates the task of meeting the learning needs of all students. Our teaching methods and materials have to adapt to these differences.

3. Learning Pace and Style

As well as differing in skill levels, students differ in the rate at which they progress in their learning and understanding. Some students have difficulty concentrating, or have a limited attention span. Students tend to prefer a particular approach to learning; some prefer learning alone, moving along at their own pace, whereas others prefer to learn in a group, sharing ideas and discussing their work. A student who prefers to work individually (or whose only experience with maths has been learning this way) may have difficulty adjusting to cooperative activities, and may need extra encouragement, support and time.

4. Reasons for Attending

Students come to class for a wide range of reasons, with different expectations and goals. These can range from a general desire to improve themselves or help their children, to preparing for further study or a specific job selection test.

Some students come ostensibly to develop their skills but the hidden agenda may include a need for social contact or friendship that is more important than the actual content of the class.

5. Students' Expectations of the Content of Maths Classes

Students attend class with a range of expectations of what they will be taught, and how. These expectations depend on their previous classroom experiences and are influenced by students' social and cultural backgrounds.

6. Differences in Attendance

Absences, lateness, students leaving the program and new students joining can disrupt the continuity of a numeracy class.

Adult students (particularly women), have many family responsibilities which sometimes interfere with regular attendance, or cause them to be late.

Ongoing enrolment, where new students join the class throughout the program, is often a feature of numeracy classes. Some numeracy teachers may thus find themselves with a different group of students at each class!

Strategies for Meeting Individual Needs

It is clearly unrealistic to expect all students to fit exactly into one learning program and all make progress at the same rate. Flexibility is the key to making an appropriate response to the wide ranging expectations and needs of students. If you are working from a course outline you will need to make changes to content, structure and teaching methods as the program proceeds.

There are a number of specific strategies that experienced numeracy teachers have used to accomodate the wide ranging needs of students. These are outlined in Handout 4.11A *Strategies for Meeting Individual Needs*.

The Activity

One effective way of dealing constructively with this issue is for participants to share their experience of individual differences and the strategies they have found personally useful. **Activity 4.11** ***Meeting Individual Needs*** is an activity in which participants brainstorm strategies for meeting individual needs. The session leader can use Handout 4.11A *Strategies for Meeting Individual Needs* to provide more ideas for discussion.

Activity 4.11 **Meeting Individual Needs**

Rationale

Dealing appropriately with individual differences is a challenge to all teachers, new and experienced alike. A useful way of finding some solutions to this issue is for participants to discuss strategies they have used, and generate new ideas in small groups. These ideas can then be shared with the group as a whole.

Aim

To develop strategies for improving the extent to which individual needs are met in numeracy programs.

Materials and Equipment

- Whiteboard or butchers' paper
- Markers

Time 30 minutes

Handouts

- Handout 4.11A *Strategies for Meeting Individual Needs* (1 for each participant)

Preparation

Read the background notes which you may wish to photocopy and distribute after the first part of the activity.

Photocopy Handout 4.11A *Strategies for Meeting Individual Needs.*

Conducting the Activity

Referring to the background notes, speak briefly about the issue of meeting individual needs.You may like to cite some examples from your own experience to stimulate thought and warm up the group for the activity which follows. Then ask participants for examples from their experience.

Questions you could ask include :

- *Who has a range of student abilities in their group?*
- *How else do students differ*? (e.g. cultural background, life experience)
- *What difficulties do these differences create for the teacher, and the students?*

Record the main ideas on whiteboard or butchers paper. At this point you may wish to distribute the background notes.

Explain to participants that the purpose of the session is to share, discuss and expand the range of effective strategies we can draw from for accomodating individual needs.

Ask participants to form groups of four to six, and discuss strategies they have used with their students to accommodate individual needs. Each group is to produce three suggestions for reporting back.

Allow ten to fifteen minutes for the discussion.

Discussion

Reconvene the entire group and ask small groups to report back. Discuss and record their strategies on butchers paper or whiteboard. If the discussion stimulates ideas, add these to the list of strategies. (Arrange to have the suggestions photocopied and distributed if possible.)

Distribute Handout 4.11A *Strategies for Meeting Individual Needs*. Discuss and evaluate any strategies that have not already been mentioned. Conclude the session by asking for feedback and comments:

- *Was the session useful? How?*
- *What new ideas did you gain about meeting individual needs?*

Strategies for Meeting Individual Needs

Listed below are some strategies which may be used to assist teachers and tutors to meet the individual learning needs of students. A range of strategies is provided, reflecting that certain approaches work well with different groups at different times. For example you may group students according to skill and ability levels for some activities, but use mixed ability groups for others. This list is neither comprehensive nor complete; rather it provides a range of ideas, some of which may be useful with your students.

Differences in Ethnic, Cultural or Political Background and Values

- Avoid culturally biased materials. If such material is unavoidable, or comes up as a natural outcome of class discussion, make sure that it is explained.
- Acknowledge cultural differences in the class and make use of them, for example by discussing different techniques of calculation, or sharing favourite recipes, or using a map to locate students' birthplaces.

Differences in Skills Levels and Learning Styles

- Use a variety of teaching strategies and tasks to suit the needs of individual learners. Include whole group, small group and individual activities, hands on materials and visual aids.
- Divide the class into small groups of students at a similar level working together, or students with a similar goal, such as passing a selection test.
- When working on a particular topic have available a range of worksheets and activities which treat the topic at different levels. Students can then work at a pace and level that suits them.
- Use self paced teaching materials or individualised worksheets, supplemented with group activities. For example, start the session with a group activity, game, or discussion which involves everyone before breaking into small groups or individual programs. In this way students work at their own pace but also have opportunities for interaction with others.
- Use 'core' and 'extension' questions on teaching materials and worksheets.
- Have worksheets or problem solving activities to challenge or extend students who have completed set work. These can be on worksheets, cards, or posted on the walls.
- If students have finished a task, encourage them to review previous work or return to an ongoing task such as learning multiplication and addition facts. Have a range of activities, games and resources to use, such as dice or cards for generating numbers to practise basic operations.

- Encourage students who have completed set tasks or make up their own problems or quizzes and swap them with each other.
- In a group where some students have reading difficulties, group students so that readers can assist with reading instructions and information.
- Use cross tutoring. More advanced students may enjoy tutoring other students. Take care that this strategy is not over used or that 'tutors' do not confuse or intimidate their 'students'.

Differences in Students' Expectations and Reasons for Attending

- Discuss students' expectations of classes openly and if necessary, negotiate individual goals and work programs.
- To avoid a totally individualised approach make sure these are used alongside group activities.

Irregular Attendance and Changing Class Numbers

- Make sessions self contained, with activities which can be completed within the time of the session. Students who miss classes can then rejoin the group without being disadvantaged.
- Encourage students to welcome latecomers and those returning to class after an absence, so that they still feel accepted by the group.

Further Ideas:

Chapter 5. Language and Maths.

Mathematics as Part of Language

There are two aspects to the relationship between mathematics and language. First, that mathematics is an important part of language and secondly, that language plays a vital role in mathematics. We have argued throughout this publication that mathematics and numeracy skills have to be seen as a vital part of Australian culture and life, that is, that mathematics needs to be seen as a part of everyday language and information. Maths is embedded in other, wider contexts, and is intertwined with at least one aspect of language - listening, reading, speaking or writing. Maths skills are required to interpret and make sense of the world. Mathematics should be taught, not in isolation, but as part of other contexts and applications.

Numeracy therefore needs to be seen not only as dealing with numbers and calculations, which are vital skills in applying mathematics to everyday situations, but needs to be concerned with the language and interpretation of mathematical data and information, and the knowledge and appreciation of size, shape, measurement, etc. Adults need to be able to analyse and interpret a variety of recorded information in tables, graphs or charts in sources such as newspapers, directories, maps, and information leaflets. The ability to talk about, and describe, the names and properties of different shapes, to follow and give directions, or to measure, are all important everyday skills that require mathematics being seen as part of our language and culture.

The teaching of adult numeracy and basic mathematics needs to be integrated with literacy and language teaching. No matter whether the teaching is part of literacy or numeracy tuition, numeracy needs to be taught within a language based context.

Language as Part of Mathematics

Language and literacy skills also play an important role in the learning, and therefore teaching, of mathematics. We have argued that in learning mathematics part of the initial process of using hands-on material, working with physical objects and/or pictures, students must talk and reflect about the experience and the concepts and processes involved. That is, there is an explicit place for language in the learning of mathematics.

Adult numeracy students, no matter what their personal level of maths may be, start from a position of knowledge of real, social and practical maths situations. This involves the use of natural language to describe the knowledge, use and application of these skills.

If students are going to learn new skills and be able to generalise and apply these to new situations they need to build upon their natural language and skills and experience more formal and structured mathematics through activities and interactions with other students and the teacher. Students need to become familiar with the ways of representing and reporting their mathematics - the ability not only to verbalise, but also to write maths, often using symbols, and to be able to read and interpret mathematics.

It is important for students to progress to an acceptance and understanding of the mathematical concepts, methods and language associated with the level of maths at which they are working. Only then can students move forward in the learning of mathematics - to be able to generalise, to use symbols and to apply the mathematics to new situations, that is, to think mathematically and to gain meaning from mathematics. This process should apply in teaching all numeracy or maths students, across the whole spectrum of mathematical skills that occur within adult basic education.

We are not arguing that all students need to learn formal 'school' maths such as algebra and solving equations. There is a common misconception that when talking about mathematics it only refers to higher level, more complicated and abstract mathematics. Mathematics refers to all levels of mathematics, from basic numeracy through to tertiary level studies. The need to generalise, or to think abstractly, in maths applies at all levels of numeracy or mathematics.

An illustration of this learning process could be with a numeracy student who started off with quite good everyday maths skills and could perform the four operations to whole numbers and money. A skill such a student may be interested in developing could be the ability to determine the 'best buy' when confronted with items in different sized containers. This student would need to be shown how to apply the operations of division and multiplication to a new situation; how these concepts relate to making the two items comparable. This requires students to move from their prior natural language and knowledge to looking at the more formal concepts of division and multiplication and then how these concepts can be generalised. Hence when confronted with the situation in the supermarket of comparing prices, students can now think mathematically and gain meaning from mathematics by applying their skills to the new situation.

The ability to think mathematically involves making decisions about how to solve a problem. It is a decision making process that may then lead to a series of actions such as calculating the solution there and then, or resorting to other strategies such as pen and paper, calculator, diagrams or hands-on materials. This 'in the head' process continues to analyse and reflect upon the problem and impacts on the approaches being taken. It requires students to be able to think mathematically - to recall or visualise previously known and relevant experiences, knowledge, words, symbols, diagrams or pictures, and to mentally manipulate these. This ability to think mathematically and visualise will not occur easily if students do not have the language or words to think with. Students need to see how their maths experiences can be represented in a visual way either through diagrams, pictures, symbols or words. Students need to feel comfortable with mathematical language in order to be able to visualise, or to think mathematically 'in the head'.

Adult numeracy students often display an inability to solve maths problems, frequently blaming a 'mental block' about maths and numbers, or else they attempt solutions but get incorrect answers because they have learned an inappropriate method (usually by rote). It is possible to see that not only have such students never really understood the processes or concepts involved, but that frequently the key to this has been their lack of understanding of the language being used.

Students will be unable to understand or apply maths if they have not understood or accepted the maths language being used throughout the process of learning their mathematics. The language of maths, which includes talking, reading, and writing maths, is a vital key to people successfully learning about, and applying, mathematics.

The Difficulties of Learning the Language of Maths

Nerida Ellerton, a lecturer in mathematics education at Deakin University, supports the view that language is a key to learning mathematics. She claims:

> *Teachers have often assumed that incorrect solutions to problems have arisen from a lack of understanding of mathematical concepts or a deficiency in computational skills. In fact, the errors have been caused by an inadequate understanding of the language of mathematics* (Ellerton, 1989, p. 95).

There are a number of difficulties associated with the learning of the language of mathematics.

Meanings of maths words

One difficulty is that students can be confused about the way we use natural, everyday words in a different way in maths. There are many examples of student confusion about the meaning and usage of words in the UK study *Children's Understanding of Mathematics:11-16*. One student when interviewed about volume responded:

Interviewer: *Do you know what volume means?*
Child: *Yes.*
Interviewer: *Could you explain to me what it means?*
Child: *Yes, it's what is on the knob on the TV set.*

(Hart, 1981)

In his analysis of maths words John Munro, who lectures in special education at Victoria College, describes three categories: (Munro, 1980):

(i) Words that maintain the same meaning in both maths usage and in everyday usage: e.g. add, take away.

(ii) Words that only have a mathematical usage and are not used in everyday contexts: e.g. numerator, quotient, isosceles, binomial, equilateral.

(iii) Words that have different meanings in their maths usage and their everyday usage. Often their meanings can be related in some way, but the maths usage is very precise e.g. volume, odd, even, square, negative, positive, factor.

Unfortunately most maths words fall into the latter two categories. The second category often involves complicated, sophisticated words. The third category can cause confusion between the different everyday and mathematical meanings and usage (see 'volume' example above). A further complication occurs when a maths word may sound the same as another word and can therefore be misheard. For example, the words 'sum' and 'some' sound the same but are very different in meaning. Is is possible for a student to mishear 'What is the sum of these numbers?' as 'What are some of these numbers?' These issues will be even more important to students with literacy problems or students from a non-English speaking background.

Words for the four main operations

An analysis of words and phrases used for the operations of addition, subtraction, multiplication and division indicates how complicated it is for adults to solve problems and interpret real life situations. Terms can overlap between different operations. For example, the phrase 'how many' is commonly used to indicate division as in 'how many fives in 25?'. But what about 'how many are there between 5 and 25?' or 'how many are five 25's?' or 'how many is 5 and 25?'. 'How many' can be used for any operation, but many students recognise it as division.

Another complication is the multitude of different words used for the one operation. Taking subtraction as an example, the common words used would include: from, minus, take away, and subtract. But what about: difference between, less, reduce, remove, decrease, discount, take off, and various other phrases that call for the use of subtraction?

An added complication with the operation of subtraction is that when solving a sum such as 13-5, the common use of 'from' requires the student to work from the second number back to the first: 'five from thirteen'. This needs to be compared to the other common terms used - take away, minus and subtract.

These all require working from the first number to the second: 'thirteen take away five' or 'thirteen minus five' etc. What happens when an unsure student suddenly moves from one teacher who used 'from' to one who uses 'minus'?

This highlights the need for teachers to be aware of the different ways words or phrases are used, and to make sure that students are equally aware of the different meanings and can acknowledge and understand the differences - that there is not just one correct way or term to be used.

Reading in mathematics

A related problem in the teaching of mathematics is that the language usually used in text books (and spoken by many teachers) is formal and sophisticated. The vocabulary of mathematics often contains words that are peculiar to mathematics and not normally used elsewhere, or words that have one meaning in the maths context and a different meaning elsewhere (mainly types (ii) and (iii) above). But it is not only the maths words that can cause the problem, often the structure of the sentences and the way they are phrased adds difficulties. Here is an example from a popular school text book (Lynch et al, 1980):

> *The lowest common denominator (LCD) is the denominator which contains a representative of factors of each of the denominators. To include all factors take the highest power of each different prime factor present.*

Analysis of other text books will provide similar examples. It is also common for text books to include such phrases as 'determine the solution to' or 'evaluate the commission on', all of which could be written in 'plain English' using terms such as 'find' or 'work out'.

Two American linguists, Dunstan Hayden and Gilberto Cuevas, have produced an analysis of some common mathematical terms in basic mathematics, and in discussing some of their concerns over the complications of maths language report:

> *We believe one should use a generic term unless a specific term has a specific mathematical content and brings clear benefits to the user. Thus we prefer to* ***change*** *a fraction to lowest common terms rather than to* ***convert it*** (Hayden and Cuevas, 1990 p.12).

UK educator Pamela Liebeck sees learning as taking place in a four stage sequence: Experience; Spoken Language; Pictures; and written Symbols; (referred to as ELPS). She makes the point that a maths textbook, no matter how good, can only be concerned with the last two stages, pictures and written symbols. The other vital two stages, experience and spoken language, cannot be included in a text book, and are dependent on the teacher introducing them to students (Liebeck, 1984).

Students will be expected to read and understand written mathematical explanations and problems - either in an everyday, real form or in a mathematical form. With most numeracy students the language faced will be about everyday situations, and formal definitions such as that quoted above should not be relevant.

Similarly the reading and writing of symbolic statements will usually be concerned with representing everyday situations and as such be familiar and meaningful within that framework.

The need to read and write more complex written and symbolic statements will apply most frequently and be more important for the higher, more sophisticated levels of adult numeracy/basic maths. Not only will the vocabulary present difficulties as outlined above, but the contexts, form and grammatical structures of the mathematics genre requires sophisticated, analytical reading and writing skills. A more detailed analysis of the grammatical skills required in mathematics has been done by Mollie MacGregor, a lecturer in mathematics education at Melbourne University, in *Reading and Writing in Mathematics* (MacGregor, 1990, pp.100 - 107) and *Reading and Writing Mathematics* (MacGregor, 1989, pp.153 - 161). In particular she argues that the use, and therefore the understanding, of prepositions such as to, from, on, of, etc. are crucial in mathematics. Another analysis by John Munro, looks in more detail at the reading of symbolic statements (Munro, 1989). The area of reading in mathematics is a major area of difficulty with students from a non-English speaking background.

To overcome maths language problems, especially with students who have literacy difficulties and with students from a non-English speaking background, maths teachers and tutors need to be able to support their students in accepting and learning mathematical language and translating it into the students' own meanings and understanding.

Teaching Strategies

In an article entitled *The Mathematics-Language Connection,* Noelene Reeves, a literacy and language educator from Western Australia, argues that 'mathematics education ... is fundamentally a language activity'; that students must be doing and talking maths in the context of an appropriate activity. She believes that student and teacher talk are vital for the student to be able to turn the activity and the learning taking place into **thinking mathematically** (Reeves, 1990, p. 91).

'Talking maths', 'writing maths', and sharing words and meanings with other students and the teacher is the best way to overcome difficulties in comprehending maths language. There are a range of language based classroom strategies which can be easily introduced into adult numeracy teaching and that will support our students in overcoming many of the traditional barriers they have faced in their learning of mathematics.

Talking maths

Teachers should be careful to explain and discuss the usage and meaning of words and terms they introduce and use.A simple strategy that could be introduced is to have a **classroom discussion about the terminology and vocabulary** to be used prior to a new topic or skill area being started.

Spend time discussing the words used - students are often fascinated by the relationships between the maths applications of words compared to their normal usage. This enables students to relate any prior knowledge of these terms, either everyday or recalled school maths knowledge, to the ways they may be about to be used in the maths classroom.

As the U.K. researchers concluded:

> *Perhaps we should get away from 'I'll show you' and into 'let us discuss what this means'* (Hart, 1981, p. 214).

- Ask students to **verbalise their actions and thought processes when solving maths problems.** It is then possible to pick up how much they have understood or their misunderstandings and misconceptions. Analysing their errors with them is a very powerful tool. However, students often find it difficult to verbalise their maths processes and thoughts. Teachers need therefore to encourage students to talk and to pose questions such as 'How did you get this?', 'What did you say to yourself when you ...?' or 'What does this really mean?'.
- It is best if the teacher can **determine what words or processes the student is familiar or comfortable with** and concentrate on using those with that particular student. From that point it is possible to extend the student's knowledge and understanding.
- Get students to **work cooperatively together** to encourage discussion, and hence help students learn and understand maths from each other. Activities such as cooperative logic problems are useful strategies to encourage classroom interaction and discussion.
- Ask students to **read out their maths work** to other students or the teacher, whether it be at the level of reading out numbers or simple word problems, through to reading out an algebraic equation.

Reading and writing

- With most numeracy students the **language presented should be kept to everyday situations,** and formal mathematical definitions can be avoided or rephrased in plain English. Where possible use generic terms rather than specific terms. The reading and writing of symbolic statements can usually be kept simple. In all cases students should be made familiar with the language being used - with the meanings being used in the mathematics context, whether via symbols or words. Students should be asked to write problems out from symbols and numerals to words, and vice versa. For example:

  $6 + 18 = 24$ . . . . . . six and eighteen make twenty four

- An important and vital starting point in writing and reading in maths is in students' ability to **read and write** numbers, especially large numbers. This relates back to their understanding of place value. Students need to be given practice in reading and writing numbers, from numerals to words and vice versa. This is necessary for both whole numbers and decimals.
- Ask students to **write in their own words** the processes or actions they have taken to solve a problem - to write down their 'in the head' calculations.

Students can be asked to keep a maths journal or diary to write about, and reflect upon, their learning.

- Writing activities can also be based around the students making up their own maths problems, for example, students could work together to create new cooperative logic problems.

Arranging cards

- Produce a series of **cards** based on some key numbers, the four basic operations and some common shopping items. The numbers may even need to be represented on the card by dots for students who need to develop their counting skills, and need to be in numerals (numbers) and written as words. The four operations and the 'equals' sign should be shown in symbols as well as written in words. These cards can then be used for students in a variety of ways: they can simply be used for recognition and counting; for matching of numbers, numerals and written numbers; or they can be used to construct word problems.

For example:

eight	and	four	makes	twelve
8	+	4	=	12

- This approach can be extended to more complex situations by increasing the scope of the words and materials represented on the cards. Items from advertising leaflets could be pasted onto cards along with their prices, and the range of words and phrases used for the four operations could be extended to include words like 'total' and 'change'. If other common key words and phrases are then incorporated, such as 'how much', 'it costs', 'what is the', 'if I buy', etc. then students could create problems like:

It costs ?	**to buy**	**loaf of bread**	**and**	**1 container of margarine**

What is the	**change**	**from $5**	**if I buy**	**a loaf of bread**	**and**
1 container of margarine					

After creating the problems the students can read them out aloud, write them out, then use the other cards to write the problem in maths symbols.

- More advanced students may not need the cards with the words or symbols on them. They can simply use the cards with the advertising material and create their own word problems to solve. This format can also be used to create problems about the metric system as many food advertisements also state their weight or volume.

The strategy with cards can be extended to much higher level mathematics. For example, it can be applied to the reading and writing of algebraic equations - a skill that many students have difficulty with. It is highly relevant to non-English speaking students.

Filling in the gaps

- Another useful technique is to use completed word problems and to white out key maths words or the numbers or amounts involved. Students can then be asked to **write appropriate words in the blank spaces**. There will often be more than one correct answer. This is similar to the cloze technique used in literacy teaching, where the strategy is used to help students to predict in reading. The word problems presented to the students should be set in familiar contexts, and the maths involved should be comfortably within their grasp.

  For example: The problem above from the cards could be presented to students as:
  It costs $ ____ to buy a loaf of bread at $1.55 and 1 container of margarine at $1.35. The _____ from $5 for the loaf of bread and the margarine was $ ____.

Other ideas

- Present a range of materials, for example a breakfast cereal box, a map, a newspaper article, a bill or invoice, with maths in it, to students. Ask them to **brainstorm the mathematics words,** numbers and symbols that are involved. This could then lead to a range of further mathematics activities and skill development depending upon the outcomes of the brainstorm and the needs and interests of the students.
- Analyse and interpret a variety of **recorded information in tables, graphs or charts**. Examples can be found from sources such as newspapers, directories, maps, and information sheets from Post Offices, Telecom, banks, etc. Ensure that students have practice in recording mathematical information in all its different forms. Students need to gather information themselves and record their information in these formats.
- Introduce students to the **appreciation, knowledge and language of shapes.** Students can be introduced to the names and properties of different shapes in a number of ways: through attribute blocks; as part of work on areas and perimeters; or through an analysis of common, everyday things such as signs and buildings. Attribute blocks are particularly good for use with students from a Non-English speaking background.

The analysis of shapes and their names can lead to a study of the related numerical latin and greek prefixes, with students using their dictionary skills to come up with a list of words starting with these prefixes.

- Use **cross number puzzles**, but use words as the questions and/or the answers.
- Use **word searches** where the hidden words are maths words that are currently in use by the students.
- Produce lists of the students' current maths words (i.e., the words they are using in their maths class) and get students to place **the words in appropriate sentences**. Students can do this from scratch or they could fill in the gaps in sentences written by the teacher.
- Look at the **mathematics used in books** - novels, short stories, travel books, diet or nutrition books, cookery books, children's books. This idea has been well documented for use with children's books by Margaret Clyne and Rachel Griffiths in *Books You can Count on: Linking Mathematics and Literature*. (Clyne and Griffiths, 1988)
- Look at commercially available **puzzle books,** some of which include a range of puzzles using numbers, shapes, maps, etc.
- Many of the earlier activities and handouts in this publication relate to maths and language, including maths involved with peoples' lives, newspapers, supermarkets, etc. All of these can be fruitfully used to encourage the learning of the language of mathematics.

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The Activities:

Activity 5.1 ***Integrating Literacy and Numeracy*** looks at how a literacy teacher/tutor could develop a range of maths activities out of everyday literacy resources.

Activity 5.2. ***Words for the Four Operations*** looks at the variety of words and expressions that are used for the operations of addition, subtraction, multiplication and division, and how these could be used with students.

Activity 5.3 ***Mathematical Talk*** looks at a range of mathematical materials or objects and focuses on the language that we use to talk about them - to describe them, to compare them.

Activity 5.4 ***Maths Words*** is an activity that looks at a range of maths words and analyses how they are used and applied in an everyday sense as well as in a mathematics sense.

Activity 5.1

Integrating Literacy and Numeracy

Rationale

This activity utilises a range of literacy and language based items such as newspapers, information leaflets and forms, maps, advertising brochures, recipes, tax forms, CES forms, payslips, maps and street directories, house kitchen or extension plans and supermarket products. They are used here to encourage participants to think about the range of numeracy related concepts which are embedded in such frequently used, information based materials. Many of these items have a literacy focus and are commonly used literacy teaching resources.

Aim

- To guide participants in the development of a plan of activities that incorporate both literacy and numeracy skills within a literacy context or theme.

Materials and Equipment

Collect a selection of literacy or language based items as described in the Rationale - 1 for each group of 4 participants.

Handout

5.1A *Literacy/Numeracy Plan*

Time 40 minutes

Preparation

Locate enough copies of the necessary range of materials to ensure that each group of participants is no larger than 4.

Select a sufficient variety of materials so that groups will be working on a range of different themes.

Analyse each of the materials you have chosen in advance - briefly list some of your ideas for both literacy and numeracy themes/activities based upon these materials. Have this list available to share with participants especially if not all maths or numeracy activities are picked up by the groups.

Conducting the Activity

Arrange participants into small groups of no more than 4 and give each group one of the selected items.

Ask each group to brainstorm on how they could use this material with students, in particular to decide upon a theme that could stem from the material, and that could be used as the basis for 2 to 4 weeks work (assuming 2 hours per week class time). The questions for the group to answer are:

- *What reading or writing skills are involved in interpreting or using these materials?*
- *What maths skills are involved in interpreting or using these materials?*

Once the theme is agreed, each group is to develop a teaching plan based around the theme and the material that includes both literacy and numeracy skill development. The group can extend their plan to incorporate other resources or materials. Give out Handout 5.1A and discuss the example given, or another example of your choosing.

Ask the groups to record their plan, with related activities and skills on Handout 5.1A *Literacy/Numeracy Plan.*

Discussion and Round up

At the end of the session get each group to share their results - either verbally, on butchers paper or whiteboards, or photocopy each group's sheets and distribute to all participants.

Point out that they were not expected to complete the whole exercise, but that it was aimed at getting them to think about the intertwining of language and maths skills. They could continue the plan at a later date to fit in with their own students.

Ask participants whether they have learnt anything about the relationship between language, literacy and mathematics from the exercise, either for themselves or for use with their students.

Ask participants how they could use this activity in their teaching. For example ask:

- *Could you use the plan you have just started to develop with your students?*
- *Could you adapt the exercise in some way or use a similar exercise with your students?*

Literacy/Numeracy Plan: An Activity for Teachers and Tutors

Plan a two to four week course based upon the material you have been given - use it as the starting point for a theme. Decide upon the goals you have for the students and the skills you might want to teach via the chosen theme. Look at the teaching strategies you would use, any other resources you would require, and any exercises or activities you could use.

Look at what **reading**, **writing** and **maths** skills are involved in interpreting or using the materials you have been given.

Lesson Planning Sheet

Goals	Skills	Teaching strategies / resources / activities
Example 1 To enable students to fill out a Medicare form	a. Identify the terms and key words on a Medicare form, e.g name, address, Medicare number etc. b. Write the necessary information such as ..etc	a. Collect and distribute to students blank Medicare forms and copies of a doctor's receipt /bill. Ask students to identify all areas of the Medicare forms and doctor's bill that they need for a claim. Students to work together and pool their knowledge and discuss all the information on the forms. b. Compile a class list of all the key words and terms related to the medicare form and the doctors bill. c. Make up cards with these terms etc.
Example 2 To enable students to understand and check a pay slip.	a. Identify the terms and key words on a pay slip e.g. gross, net, tax, deductions, employee, employer, etc b. Explain the meaning of these key words. c. Calculate and check their own salary.	a. Ask students to collect any pay slips they have or can get access to. White out and change any names and addresses, and then copy and distribute to students. Ask students to identify all the areas and words from the payslip. b. Present different pay rates and times, etc. and work together to explain and calculate terms such as gross salary, tax, etc. c. Get students to fill out payslips when presented with rates of pay etc.

Handout 5.1 p.1

Breaking The Maths Barrier, B. Marr and S. Helme, 1991

Lesson Planning Sheet

Date **Theme** **Class/student**

Goals	Skills	Teaching strategies /resources/ activities

Breaking The Maths Barrier, B. Marr and S. Helme, 1991

Activity 5.2 Words for the Four Operations

Rationale

The application and use of the four main operations of addition, subtraction, multiplication and division are fundamental skills in the teaching of numeracy. Yet it is often assumed that these operations, and the related words and symbols, are simple to understand and use. This exercise sets out to highlight the wide range of words and phrases that are commonly used in the application of the four operations. It also gives participants a hands on opportunity to develop a strategy for teaching the application of the four operations to everyday situations.

Aims

- To develop teachers' awareness of the range of terms used for the four common maths operations.
- To provide an opportunity for participants to work together to develop some practical strategies and materials for using with students.
- To provide participants with strategies for remembering and analysing the terms and phrases used for the four operations of addition, subtraction, multiplication and division.

Materials and Equipment

- Pieces of paper or card, scissors, textas and glue
- Advertising brochures.

Handouts

- 5.2A *Symbols and Words*

Time

30 minutes

Preparation

Photocopy Handout 5.2A, one copy for each participant. If necessary, prepare an OHP transparency of Handout 5.2A for the summing up.

Collect materials suitable for pasting onto cards: i.e. pictures of supermarket hardware, large retail store advertising brochures; the card, scissors, textas and glue. The cards will need to be about 8 cm by 20 cm.

Prepare one set of cards to show as an example - see example below. Use an advertising brochure to cut out pictures of two items such as a packet of margarine and a loaf of bread.

Conducting the Activity: Part A

Give each participant Handout 5.2A.

Ask each participant to fill in each of the boxes with any words or expressions they know which indicate that they should perform the stated operation, or that mean 'equals'. To get them started give them some common terms such as 'add' and 'take away', as well as indicating that they should look for everyday expressions such as 'total' or 'deduct'.

For an even easier start you may want to do the addition together, alternatively, the entire list could be compiled by brainstorming as a group.

The table below lists some examples of these terms or phrases.

Operation Symbol	Some examples of possible key words and phrases
+	Plus, add, sum, total, altogether, more, increased by, credit, deposit
–	minus, subtract, difference, difference between, less, take away, from, withdraw, debit, reduced by, discount, off
x	multiply, product, times, of, lots of, groups of, by, 's (as in seven five's)
÷	divide, quotient, into or goes into, how many ... 's in ...?, share
=	equals, is, are, makes, the answer is

Allow sufficient time for participants to fill in the sheet. Collect all the responses onto a common list on a whiteboard, or an OHP.

Discussion of Part A

Discuss the complexity of the range of terms collected. Use questions like:

- *Are maths terms well defined and succinct, or are there difficulties when interpreting and applying even the four simplest maths operations?*
- *How would literacy students cope with this range?*
- *How could you use this exercise with students?*

List ideas on the board or OHP.

Introduce the next part of the activity - using the table to make cards.

Part B

Group participants in small groups of 4 to 5.

Explain the activity first:

> *In this part we are going to make cards from advertising material. Use your words from the symbol table to make up sentences that represent an everyday maths application. One such example is*

How much	**would it cost to buy**	**a loaf of bread**
and	**a packet of margarine**	

This example is made up of word cards saying 'how much', 'would it cost to buy', 'and', as well as pictures of a loaf of bread and a packet of margarine.

Give out advertising materials, cards, pens, glue and scissors to each group. Ask participants to cut out pictures of items from the advertising materials and paste them onto the cut up cards. They also need to write other maths words or phrases onto the cards from their symbols table. Using their cards, each group is to make up sentences that represent an everyday maths application. They are to use the words or terms from the table as well as other terms they find are necessary to complete a sentence.

Discussion of Part B

Ask for any comments about the use of cards in this way with numeracy students. Discuss any suggestions about different ways of using the concept, especially in terms of different skill levels of students.

Symbols and Words

Operation Symbol	Some examples of possible key words and phrases
+	
-	
x	
÷	
=	

Activity 5.3 Mathematical Talk

Rationale

Mathematical language is frequently used in everyday situations to label and classify, to compare and order, or to change and modify objects. Many students will in fact have much of this language, but will not recognise it as being mathematical. This activity is designed to explore the wide range of words that are used to describe, order and change objects and to categorise these words.

Aim

To develop teachers awareness and knowledge of the breadth and scope of mathematical language, in particular to look at what words we use to:

- group and name;
- order and compare;
- change and transform.

Materials and Equipment

A range of sets of common items as described in Preparation. Prepare one set of two or more items for each group of participants.

Possible items:

- 2 or more different packages e.g. breakfast cereal boxes, shampoo bottles, paint tins.
- different pieces of fruit
- copies of different road signs
- pieces of timber
- pieces of sewing material
- sets of coins

Handouts

- 5.3A *Mathematical Talk*

Time

30 minutes

Preparation

Collect a range of items to be made into sets of two or more. Each set should be related but have different attributes. They need to incorporate some maths characteristics such as size, shape, number, and symbols.

Locate enough of the necessary range of materials to ensure that each group of participants has one set of items and there are no more than four in the group.

Select sufficiently different types of materials so that groups will be working on a very different variety of objects.

Photocopy Handout 5.3A, one for each participant. If necessary, prepare an OHP transparency of Handout 5.3A for the summing up.

Conducting the Activity

Group participants into small groups of no more than 4 and give each a prepared set of items.

Ask each group to brainstorm how they can talk about, describe and compare the items in front of them. What maths words, numbers or symbols are displayed on the items or are concerned with the items? The group could appoint a recorder to list all the maths related words, numbers, or symbols used.

Distribute Handout 5.3A *Mathematical Talk.* The group is to then record their numbers, words or expressions on the handout under the three categories.

1. **Grouping and naming.**

- *What words or symbols do we use to group, sort, classify, label and name?*
  e.g. small, large, box, litre, 200 ...

2. **Ordering and comparing.**

- *What words or symbols do we use to talk about the relationships between groups or things; to compare, order, distinguish or categorise?*
  e.g. heavier, more, first, rounder, same ...

3. **Changing and transforming.**

- *What words or symbols do we use to talk about changing or transforming items or groups - to add to or to share?*
  e.g. divide, cut, split, together, increase ...

Discussion and Round up

At the end of the session ask each group to share their results - either verbally, onto an OHP transparency, on butchers paper or whiteboards, or photocopy each group's sheets for all participants.

Point out that the aim was to get them to think about the intertwining of language and maths skills, and that there are many, often everyday, numbers, words and symbols used to describe and compare.

Ask participants whether they have learnt anything about the use of mathematical language from the exercise, either for themselves or for use with their students.

Ask participants how they could use this activity in their teaching. For example:

- *Could you adapt the exercise in some way to use with your students?*
- *Would it help you to find out the breadth of maths knowledge and language of your students?*
- *Could the idea be used as the the basis for writing as part of numeracy and literacy skill development?*

Mathematical Talk

The aim of this exercise is to think about the range of maths related words that are used to describe and compare common items. Based upon the items you have been given, list the words you have thought of under the following three headings.

1. Grouping and naming:

What words do we use to group, sort, classify, label and name?

2. Ordering and comparing:

What words do we use to talk about the relationships between groups or things, to compare, order, distinguish or categorise?

3. Changing and transforming:

What words do we use to talk about changing or transforming items or groups - e.g. to add to or to share?

Maths Words

Rationale

There are a number of difficulties associated with the language of mathematics. One is that people can be confused about the way we use natural, everyday words in a different way in maths. This activity provides participants with the opportunity to explore for themselves the meanings of a range of maths words. Many literacy teachers may be unfamiliar with some of the words or their usage in maths, and this activity will help them to learn their meanings either from other participants or from dictionaries.

Aims

- To develop participants' awareness of the breadth and scope of mathematical language.
- To provide an opportunity for participants to analyse and become aware of the differences in meanings of words used in an everyday sense.

Materials and Equipment

- Dictionaries - both general and mathematical

Time

30 minutes

Handouts

- 5.4A *Maths Words*
- 5.4B *Lists of Maths Words*

Preparation

Photocopy Handout 5.4A, one for each participant. Also photocopy Handout 5.4B and cut up enough lists of words to give one of A, B or C to each group of four participants. If necessary, prepare an OHP transparency of Handout 5.4A for the summing up.

Conducting the Activity

Arrange participants in small groups of no more than 4 and give each group a list of maths words, and each participant a copy of Handout 5.4A.

Ask each group to discuss the meaning of the words on their list, especially in relation to the three categories of words listed on Handout 5.4A. Refer them to the available dictionaries if necessary.

Then ask the group to record their words under one of the three categories on Handout 5.4A.

The small group discussion will hopefully highlight the confusion between the everyday usage and the specific maths usage of words. It may also highlight literacy difficulties with some of the more sophisticated maths terminology.

Discussion and Round up

At the end of the session get each group to share their results with other groups - either verbally, onto an OHP transparency, or on butchers' paper or whiteboards. Use this summing up session to discuss any words which are more difficult to categorise.

Ask participants for their reactions about the activity, and any difficulties with some of the sophisticated maths terminology.

Ask participants what the implications of this activity are for their work with numeracy students.

Point out that this approach is one that they should put into practice with their students to check the knowledge and understanding of the related maths language of their students. In particular it can be used with students to highlight the differences between everyday uses of the words, with which they may be familiar, and their specific maths usage. At the start of a new topic have a discussion with the students about the meanings of any maths vocabulary they are about to meet, or, as words with different meanings emerge make sure they are discussed by the group.

Maths Words

Place each word on your list into ONE of the following categories.

Category 1: **Words that maintain the same meaning in both maths usage and in everyday usage:**

Category 2: **Words that only have a mathematical usage and are not used in everyday contexts:**

Category 3: **Words that have different meanings in their maths usage and their everyday usage: (often their meanings can be related in some way, but the maths usage is very precise.)**

Handout 5.4B

Lists of Maths Words

LIST A

- Add
- Multiple
- Square
- Consecutive
- Average
- Product
- Odd
- Degrees
- Graph
- Operation
- Isosceles
- Ratio
- Prime
- Horizontal
- Parallel
- Take-away
- Sum
- Integer
- Irrational
- Pentagon

LIST B

- Multiply
- Denominator
- Scalene
- Positive
- Percentage
- Subtract
- Fraction
- Set
- Cancel
- Quarter
- Difference
- Volume
- Cube
- Factor
- Radius
- Even
- Times
- Circle
- Digit
- Remainder

LIST C

- Divide
- Metre
- Plane
- Centre
- Triangle
- Minus
- Perimeter
- Adjacent
- Mean
- Trapezium
- Quotient
- Area
- Negative
- Estimate
- Equivalent
- Equilateral
- Kilogram
- Reciprocal
- Tables
- Total

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V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:

Acquisitions Coordinator
ERIC Clearinghouse on Adult, Career, and Vocational Education
Center on Education and Training for Employment
1900 Kenny Road
Columbus, OH 43210-1090

If you are making an unsolicited contribution to ERIC, you may return this form (and the document being contributed) to:

(Rev. 9/91)